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PROCEEDINGS.

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1923.

THE PHYSICAL SOCIETY OF LONDON.

1923-24.

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PROCEEDINGS

AT THE

MEETINGS OF THE PHYSICAL SOCIETY OF LONDON.

SESSION 1922-1923.

Meetings held at the Imperial College of Science,

The Chair being taken at all the meetings by the President, ALEXANDER RUSSELL, M.A., D.Sc.

October 27, 1922.

1. The Presidential Address was delivered by ALEXANDER RUSSELL, M.A., D.Sc.

2. A Demonstration of A Radio-Telegraphic Device was given by T. D. PARKIN, of Marconi's Wireless Telegraph Co., Ltd.

3. A Paper on "The Problem of Two Electrified Spheres" was read by ALEXANDER RUSSELL, M.A., D.Sc.

November 10, 1922.

The following Papers were read :—

1. "The Homographic Treatment of the Symmetrical Optical Instrument," by G. TEMPLE (Birkbeck College).

2. "On the Structure of the Sulphur Dioxide Molecule," by Prof. A. O. RANKINE, D.Sc., and C. J. SMITH, M.Sc., A.R.C.S., D.I.C.

3. "The Thermal Effect of Vapours on Rubber," by A. S. HOUGHTON, B.Sc., A.I.C.

A DEMONSTRATION of An Apparatus for Testing the Tensile Strength of Gas Mantles, was given by J. T. ROBIN.

November 24, 1922.

The following Papers were read :—

1. "The Theory of the Singing Flame," by E. G. RICHARDSON, B.Sc.

2. "Unit Surfaces," by Miss ALICE EVERETT, M.A.

3. "Vibration Galvanometers with Asymmetric Moving Systems," by Prof. R. LL. JONES.

A DEMONSTRATION of Some Applications of the Gyroscope, was given by PAUL SCHILOWSKY, Chairman of the Gyroscopic Society, Petrograd.

A DEMONSTRATION of A New Balance for Compensating the Temperature Error of Watches and Chronometers, was given by PAUL DITISHEIM, La Chaux de Fonds, Switzerland.

December 8, 1922.

The following Papers were read :—

1. "The Relation between Molecular and Crystal Symmetry as shown by X-Ray Crystal Analysis," by G. SHEARER, M.A., B.Sc.

2. "Modification of the Powder Method of Determining the Structure of Metal Crystals," by E. A. OWEN, M.A., D.Sc., and G. D. PRESTON, B.A.

3. "The Cathode Ray Oscillograph," by A. B. WOOD, D.Sc.

A DEMONSTRATION of A Low Voltage Cathode Ray Oscillograph, was given by R. WEBB, International Western Electric Company.

January 3 and 4, 1923.

The Annual Exhibition of Apparatus of the Physical and Optical Societies was held from 3-6 p.m. and 7-10 p.m.

The following Lectures were delivered each day :—

"Reproduction of Colour by Photographic Processes," by Mr. W. GAMBLE.

"Recent Photo-Elastic Researches on Engineering Problems," by Prof. E. G. COKER, F.R.S.

January 26, 1923.

The following Papers were read :—

1. "A Supposed Relationship between Sunspot Frequency and the Potential Gradient of Atmospheric Electricity," by Dr. C. CHREE, F.R.S.

2. "A Further Improvement in the Sprengel Pump," by J. J. MANLEY, M.A.

3. "Null Methods of Measurement of Power-factor and Effective Resistance in Alternating Current Circuits by the Quadrant Electrometer," by Dr. D. OWEN.

A DEMONSTRATION of An Electro-capillary Relay for "Wired Wireless," was given by Major C. E. PRINCE, O.B.E.

February 9, 1923.

Annual General Meeting.

GENERAL BUSINESS.

The Annual Report was read by the Secretary, Mr. F. E. SMITH, O.B.E., F.R.S., and the Report of the Treasurer was read by the Treasurer, Mr. W. R. COOPER, M.A., B.Sc. Both Reports were unanimously adopted.

ANNUAL REPORT.

During the year thirteen ordinary Science Meetings and one Special Meeting of the Society were held. The former were held at the Imperial College of Science, and the latter, by the kind invitation of Sir Joseph Petavel, F.R.S., at the National Physical Laboratory.

In all 30 separate Papers were presented to the Society, and there were 17 demonstrations of special devices.

The Seventh Guthrie Lecture was delivered by Professor N. Bohr on March 24, the subject being "The Effect of Electric and Magnetic Fields on Spectral Lines." The lecture was attended by 165 Fellows and Visitors.

On May 26 Dr. F. W. Aston, F.R.S., delivered a special lecture on "Atomic Weights and Isotopes," and on October 27 Dr. Alexander Russell gave his Presidential Address, in which he described recent developments in physical science, and the part the Physical Society had played therein.

Excluding the Guthrie Lecture and the visit to the National Physical Laboratory, which was attended by about 150 Fellows and their friends, the average attendance at the meetings was 72. The demonstrations during the year were undoubtedly attractive, and the Council takes this opportunity of reminding Fellows that demonstrations are welcome at all meetings of the Society. Expenses of transport of apparatus can, in most cases, be paid by the Society.

In co-operation with the Optical Society the Twelfth Annual Exhibition of Scientific Apparatus was held on January 4 and 5. Fifty-five firms exhibited apparatus, and in many cases interesting demonstrations were arranged. Two special demonstrations were given by Dr. F. L. Hopwood. The Exhibition was the most successful ever held by the Society, the attendance reaching nearly 3,000. Mr. A. A. Campbell Swinton, F.R.S., gave two lectures on the "Johnsen-Rahbek Electrostatic Telephone and Its Predecessors"; Sir Frank Dyson, F.R.S., Astronomer Royal, gave a lecture on "The Employment of Coarse Wire Gratings in Astronomy," and Mr. F. Harrison Glew delivered

a lecture on "Radium; Its Applications in Peace and War." The lectures were illustrated by experiments and were attended by large audiences. The Society is greatly indebted to the Governing Body of the Imperial College for allowing the Exhibition to be held in its rooms; it is also indebted to many members of the staff of the College for valuable assistance.

Mr. C. E. S. Phillips and Mr. F. E. Smith were appointed by the Council to represent the Society on the Council of the Institute of Physics; Dr. D. Owen and Mr. Cooper represented the Society on "Science Abstracts" Committee until October 27 last. On the appointment of Mr. Cooper as Editor of Science Abstracts, Mr. T. Smith replaced Mr. Cooper as the Society's representative.

The Institution of Electrical Engineers having reported that the financial position of "Science Abstracts" necessitated a reconsideration of the Society's contribution, a small Committee was appointed to make recommendations. As a first action Council agreed that from January 1, 1923, Fellows subscribing for Section B shall be charged ten shillings per annum; this appears to be slightly less than the cost price.

The first edition of Mr. Jeans' Report on the Quantum Theory has been sold out; there are many inquiries for copies, and Mr. Jeans has kindly agreed to revise and extend the Report. The new edition will be ready shortly. Professor Fowler's Report on "Series in Line Spectra" has already been issued to Fellows.

The question of arranging for occasional meetings of the Society in provincial towns has received careful consideration, and is viewed with favour. It is hoped to arrange for one such meeting during 1923.

In 1921 the Council informed the Conjoint Board that it would welcome any reciprocal arrangement by means of which the publications of any Scientific Society might be sold at a slightly reduced rate to Fellows of other Societies. The Conjoint Board obtained the opinions of other scientific bodies; of 40 definite replies, 16 Societies would agree to a reduction, and 24 were not in favour. No further action for the present is recommended.

Owing to some difficulty in the preparation of the Duddell Memorial Medal, the first award was not possible in 1922. The Committee dealing with the memorial have informed Council that the difficulty has been overcome, and an award will be made in 1923.

The number of Honorary Fellows on the Roll on December 31, 1922, was ten, and the number of ordinary Fellows and Students was 572. During the year 45 new Fellows and 5 Students were elected, and there were 5 resignations.

The Society has to record with regret, the deaths of Mr. C. J. Lambert, Mr. F. W. Sanderson, Mr. H. M. Elder, Professor Trouton, and Mr. L. W. Shave. Mr. C. J. Lambert was elected a Fellow during the first session of the Society, 48 years ago; he was a Life Fellow and took great interest in the proceedings of the Society. Mr. F. W. Sanderson, the late Headmaster of Oundle School, was also a Life Fellow, having been elected in 1885. Mr. H. M. Elder was elected a Fellow in 1886, and had served as a Secretary and as a Vice-President. Professor Trouton was made a Fellow in 1895 and had served on the Council. Mr. L. W. Shave was one of the youngest Fellows, having been elected but 12 months ago; he took a very keen interest in the work of the Society.

The Council have also to record, with regret, the death of one Honorary Fellow, Dr. R. Benoît, of Paris. Dr. Benoît was made an Honorary Fellow in 1909.

Report of the Treasurer.

The Accounts this year show the very unusual position of a debit balance, the expenditure having exceeded the income by £60 13s. 9d. Fortunately, this does not mean that the Society is in an unsatisfactory financial position. The deficit is due entirely to the publication of Prof. Fowler's Report on Spectra, of which the cost of printing was £358 5s. 5d. Against this cost must be set the receipts from sales of the Report, and as these will continue there is not much doubt that the present deficit will be made good in the following year.

The income has increased by nearly £120, being £1,778 19s. 8d., as compared with £1,660 5s. 4d. for the previous year. This is chiefly due to sales of publications, which realised £539 4s. 11d. The increase in subscriptions was small.

The expenditure on the "Proceedings" and the ordinary publications has decreased by £59 3s. 11d., but owing to the special publications the total cost of printing and distribution has risen to £1,256 1s. 1d., as compared with £1,095 15s. 7d. in the previous year.

An unsatisfactory feature in the Balance Sheet is a substantial increase in the "subscriptions in arrears." The investments have been valued at market prices, through the courtesy of the Manager of the London County Westminster & Parr's Bank at Charing Cross, and at length they show a marked appreciation. During the year the sum of £199 12s. 6d. was invested by the purchase of £200 5 per cent. War Loan, this investment being in respect of composition fees.

The Trust Fund of £500 London, Brighton & South Coast Railway Ordinary Stock, which was transferred to the Society by the late W. F. Stanley in 1901 for the establishment of the "Bulletin," has been set out separately so as to indicate clearly that this stock is distinguished from the other investments by being in trust.

Election of Officers and Council.

Dr. Lewis Simons and Mr. W. Shaw having been nominated scrutators, a ballot was held for the Officers and Council for the year 1923-1924. The following were elected :—

President : Alexander Russell, M.A., D.Sc.

Vice-Presidents : E. H. Barton, D.Sc., F.R.S. ; Prof. T. Mather, F.R.S. ; T. Smith, B.A. ; C. R. Darling, F.I.C.

Secretaries : F. E. Smith, O.B.E., F.R.S. ; D. Owen, B.A., D.Sc.

Foreign Secretary : Sir Arthur Schuster, Ph.D., Sc.D., F.R.S.

Treasurer : W. R. Cooper, M.A., B.Sc.

Librarian : Prof. A. O. Rankine, D.Sc.

Other Members of Council : R. W. Paul ; Prof. C. L. Fortescue, O.B.E. ; W. S. Tucker, D.Sc. ; E. H. Rayner, M.A., D.Sc. ; J. H. Brinkworth, B.Sc. ; J. Guild, A.R.C.Sc., D.I.C. ; F. L. Hopwood, D.Sc. ; E. A. Owen, B.A., D.Sc. ; J. H. Vincent, D.Sc., M.A. ; G. B. Bryan, D.Sc.

INCOME AND EXPENDITURE ACCOUNT.

From January 1st to December 31st, 1922.

Income.	£	s.	d.	£	s.	d.
Entrance Fees	44	2	0			
Subscriptions by Fellows *	728	8	11			
" Voluntary	12	12	0			
" by Students	4	4	0			
" Arrears paid	25	19	9			
" Paid in Advance	38	2	9			
" for "Science Abstracts" and Advance Proofs	30	11	9			
				884	1	2
Composition Fees				52	10	0
Sales of Publications (Fleetway Press, Ltd.)				539	4	11
Dividends :—						
Furness Debenture Stock	11	7	10			
Midland Railway	28	15	0			
Metropolitan Board of Works	7	0	0			
Lancaster Corporation Stock	8	14	0			
New South Wales Stock	8	17	10			
London, Brighton & South Coast Railway	19	9	8			
Great Eastern Railway	14	5	0			
India 3½% Stock	17	10	0			
Funding Loan	18	17	0			
War Stock 5%	5	0	0			
				139	16	4
Income Tax refunded				42	1	6
Interest on deposit account				6	14	6
Optical Society (for Exhibition)				18	6	3
Exhibitors' payments on account of catalogue				96	5	0
				1,778	19	8
Balance, being excess of Expenditure over Income				60	13	9
				£1,839	13	5

Expenditure.	£	s.	d.	£	s.	d.
"Science Abstracts" (Inst. El. Eng.)				309	4	6
Ordinary Publications :—						
"Proceedings"	568	1	11			
"Bulletin"	56	8	11			
General, including printing for Exhibition	168	4	6			
				792	15	4
Special Publications :—						
Discussion on Hygrometry	24	18	7			
Report on Spectra	358	5	5			
				383	4	0
Postage of Publications				80	1	9
Reporting	79	10	0			
Refreshments and Attendance	96	14	7			
Petty Cash—						
Secretaries' Expenses	20	18	7			
Treasurer's Expenses	20	10	0			
Guthrie Lecture (Honorarium)	10	0	0			
Report on Spectra (Honorarium)	20	0	0			
Periodicals	3	17	6			
Insurance	1	15	0			
F. S. Spiers (registered address)	2	2	0			
Brokerage and Bank Charges	1	2	2			
Advertising	7	18	0			
				264	7	10
Conjoint Board of Scientific Societies				10	0	0
				£1,839	13	5

W. R. COOPER, *Honorary Treasurer.*

Audited and found correct.

February 1st, 1923.

ROBT. W. PAUL
F. J. W. WHIPPLE } *Honorary Auditors.*

* Forty-one Fellows paid reduced subscriptions by the arrangement with the Institute of Physics, the total discount amounting to £15 17s. 11d. "Voluntary Subscriptions" are subscriptions paid by Fellows who compounded for the low sum of £10.

BALANCE SHEET AT DECEMBER 31ST, 1922.

ASSETS.

	£	s.	d.	£	s.	d.
Subscriptions in arrears	84	15	0			
Less reserve for subscriptions probably unrealisable	44	15	0			
				40	0	0
Investments (valued at Dec. 31):—						
£533 Furness Railway 3 per cent. Debenture Stock	336	0	0			
£1,600 Midland Railway $2\frac{1}{2}$ per cent. Perpetual Preference Stock	800	0	0			
£200 Metropolitan Board of Works $3\frac{1}{2}$ per cent. Consolidated Stock	186	0	0			
£400 Lancaster Corporation 3 per cent. Redeemable Stock	268	0	0			
£254 2s. 9d. New South Wales $3\frac{1}{2}$ per cent. Ordinary Stock	249	0	0			
£500 London, Brighton & South Coast Railway Ordinary Stock	405	0	0			
£500 Great Eastern Railway 4 per cent. Debenture Stock	410	0	0			
£500 India $3\frac{1}{2}$ per cent. Stock	315	0	0			
£650 4% Funding Loan, 1960-90	559	0	0			
£200 5% War Loan 1929/47 inscribed (purchased for £199 12s. 6d. in 1922)	200	0	0			
				3,728	0	0
Stock of Publications (Treasurer's valuation)				400	0	0
Cash at Bank, on Deposit	100	0	0			
Cash at Bank, Current Account	113	10	9			
	213	10	9			
Adjustment for outstanding cheques, &c.	46	14	0			
				166	16	9
Cash in hand (Treasurer's Petty Cash)				5	3	
				£4,335	2	0

W. R. COOPER, *Honorary Treasurer.*

LIABILITIES.

	£	s.	d.
Life Compositions	1,995	10	0
Balance, General Fund	2,339	12	0
	£4,335	2	0

Audited and found correct,

February 1st, 1923.

ROBT. W. PAUL
F. J. W. WHIPPLE } *Honorary Auditors.*

Note.—The value of the Society's library has not been brought into the Balance Sheet.

LIFE COMPOSITION FUND AT DECEMBER 31ST, 1922.

	£	s.	d.
130 Fellows paid £10	1,300	0	0
3 Fellows paid £15	45	0	0
1 Fellow paid £20 10s.	20	10	0
12 Fellows paid £21	252	0	0
12 Fellows paid £31 10s.	378	0	0
158	£1,995	10	0

W. F. STANLEY TRUST (FOR THE "BULLETIN").

	£	s.	d.		£	s.	d.
£500 London, Brighton & South Coast Railway...	405	0	0	Carried to Balance Sheet	405	0	0

Audited and found correct,

W. R. COOPER, *Honorary Treasurer.*

February 1st, 1923.

ROBT. W. PAUL
F. J. W. WHIPPLE } *Honorary Auditors.*

ELECTION OF HONORARY MEMBER.

A ballot was held for a new honorary member, and the following was elected: Dr. S. W. Stratton, President of the Massachusetts Institute of Technology, and formerly Director of the Bureau of Standards, Washington, D.C.

VOTES OF THANKS.

The following votes of thanks were carried unanimously: To the Hon. Auditors (Messrs. R. W. Paul and F. J. W. Whipple), proposed by Dr. Allan Ferguson and seconded by Dr. W. S. Tucker. To the retiring Officers and Council, proposed by Dr. R. T. Beatty and seconded by Mr. B. W. Clack. To the Governing Body of the Imperial College of Science, proposed by Dr. C. Chree and seconded by Dr. H. Borns.

SCIENTIFIC BUSINESS.

The following Paper was read :—

"The Eötvös Torsion Balance," by Capt. H. SHAW, M.Sc., and E. LANCASTER-JONES, B.A.

A DEMONSTRATION of The Flame Phone (Scientific and Projections, Ltd.), was given by H. W. HEATH, B.Sc.

February 23, 1923.

DEMONSTRATIONS AND DISCUSSION ON "X-RAY MEASUREMENTS."

At the AFTERNOON MEETING the following Demonstrations were shown :—

1. "Method of Measuring X-Ray Intensity," by Major C. E. S. PHILLIPS.

2. "Intermittent Discharge from Sectorless Static Machine," by E. J. EVANS, B Sc.

3. "An X-Ray Balance," by L. H. CLARK, M.Sc.

4. "Dr. Solomon's Ionometer," by H. B. GOUGH, Sunic Research Laboratories.

5. "A Spectrometer for Measuring End-radiation," by W. E. SCHALL, B.Sc.

6. "The Ondoscope," by F. L. HOPWOOD, D.Sc.

At the EVENING MEETING an introductory Address was given by Sir WILLIAM H. BRAGG, K.B.E., F.R.S., and the following Papers were then read :—

1. "The Measurement of X-Ray Intensity, and the Necessity for an International Method," by Professor SIDNEY RUSS.

2. "The Quality of X-Rays Produced by Various Types of High-tension Generators and an Incandescent X-Ray Bulb," by F. J. HARLOW, B.Sc., F.Inst.P., and E. J. EVANS, B.Sc.

3. "Practical X-Ray Measurements for Medical Purposes," by Dr. MARTIN BERRY.

4. "X-Ray Protective Materials," by Dr. G. W. C. KAYE and Dr. E. A. OWEN.

March 9, 1923.

The following Papers were read :—

1. "The Crystalline Structure of Anthracene," by Sir W. H. BRAGG, K.B.E., F.R.S.

2. "On the Frequency of Vibration of Circular Diaphragms," by J. H. POWELL, M.Sc., and J. H. T. ROBERTS, D.Sc.

3. "Radio-acoustic Method of Locating Positions at Sea," by A. B. WOOD, D.Sc., F.Inst.P., and Capt. H. E. BROWNE, O.B.E., R.N.

March 23, 1923.

The following Papers were read :—

1. "On a New Moving-coil Galvanometer of Rapid Indication," by Dr. W. J. H. MOLL, University of Utrecht.

2. "A Thermopile for Measuring Radiation," by Dr. W. J. H. MOLL.

3. "Note on Aberration and the Doppler Effect as Treated in the Theory of Relativity," by Capt. C. W. HUME, M.C., B.Sc.

The following Experimental Demonstrations were shown :—

"The Production of Electromotive Forces by Heating Junctions of Single Metals," by Mr. C. R. DARLING, F.I.C., F.Inst.P., and the Hon. C. W. STOPFORD.

"The Double Refraction due to Motion in a Vanadium Pentoxide Sol, and Some Applications," by Mr. R. H. HUMPHRY, M.Sc.

April 27, 1923.

The following Papers were read :—

1. "The Analysis of Bubbles in Glass," by J. W. RYDE and R. HUDDART, of the Research Staff of the General Electric Company, London.

2. "A Simple Regenerative Vacuum Device and Some of its Applications," by H. P. WARAN, M.A., Ph.D.(Cantab.), F.Inst.P.

3. "Application of the Eötvös Torsion Balance to the Investigation of Local Gravitational Fields," by Capt. H. SHAW and E. LANCASTER-JONES, B.Sc.

The following Demonstrations were shown :—

"An Electromagnetic Inductor," by L. F. RICHARDSON, F.Inst.P.

"An Experiment Demonstrating Time-lag in Vision," by F. LL. HOPWOOD, D.Sc., A.R.C.Sc., F.Inst.P.

May 11, 1923.

The Duddell Medal was handed over to the President of the Society by Sir William H. Bragg, K.B.E., F.R.S., Chairman of the Duddell Memorial Committee.

The Eighth Guthrie Lecture was delivered by J. H. JEANS, D.Sc., LL.D., F.R.S., who took as his subject "The Present Position of the Radiation Problem."

May 25, 1923.

The following Papers were read :—

1. "The Effect of Torsion on the Thermal and Electrical Conductivities of Metals," by Prof. C. H. LEES, D.Sc., F.R.S., and J. E. CALTHROP, B.A., B.Sc.

2. "The Use of the Wien Bridge for the Measurement of the Losses in Dielectrics at High Voltages, with Special Reference to Electric Cables," by A. ROSEN, B.Sc., A.M.I.E.E.

The following Demonstrations were shown :—

"An Experiment on the Production of an Intermittent Pressure by Boiling Water," by C. R. DARLING, F.I.C., F.Inst.P.

"A Novel Instrument for Recording Wireless Signals," by N. W. McLACHLAN, D.Sc., M.I.E.E.

June 8, 1923.

Prof. JAMES G. GRAY, D.Sc., F.R.S.E., Cargill Professor of Applied Physics in the University of Glasgow, delivered a Lecture entitled "A General Solution of the Problem of Finding the True Vertical for all Types of Marine and Aerial Craft." The Lecture was accompanied by Demonstrations with Gyroscopes.

June 22, 1923.

Prof. F. HORTON, M.A., D.Sc., F.R.S., Royal Holloway College, University of London, delivered a Lecture entitled "The Excitation and Ionisation Potentials of Gases and Vapours."

THE DUDDELL MEMORIAL MEDAL.

OBVERSE.

REVERSE.

DUDDELL MEMORIAL

William Du Bois Duddell was born in London in 1872, and was educated privately and at the College Stanislas, Cannes. He served his apprenticeship as an engineer with Messrs. Davey, Paxman & Co., of Colchester, from 1890 to 1893. He then studied at the City and Guilds Central Technical College at South Kensington from 1893 to 1900 under the late Professor Ayrton, obtaining a Whitworth Exhibition in 1896 and a Whitworth Scholarship in 1897 ; subsequently (in 1913) the Fellowship of the City and Guilds of London Institute was conferred upon him. He remained on at the College engaged in original experimental work, and his unusual talents as an experimentalist soon became apparent. During this period he developed his oscillograph for the study of alternate current wave forms. In 1898, in conjunction with Mr. (now Professor) E. W. Marchant he read a Paper before the Institution of Electrical Engineers, entitled " Experiments on Alternate-current Arcs by the Aid of Oscillographs," and it was on this occasion that he gave the first of many brilliant demonstrations on the application of this instrument. Our present knowledge of the theoretical side of alternators, switchgears, surges in cables and telephonic phenomena undoubtedly owes much to the Duddell Oscillograph. In 1900, in a Paper read before the Institution of Electrical Engineers, entitled " On Rapid Variations of Current through the Direct-current Arc," Duddell described the " musical arc " as a generator of high-frequency currents. This discovery was of fundamental importance, and the musical arc in the hands of Poulsen became the means of producing continuous electric waves for telegraphic purposes. Duddell later turned his attention to wireless telegraphy, and by his thermo-galvanometer was able to measure the currents received in the antenna circuit under various conditions. In 1909 he described before the Physical Society his bifilar vibration galvanometer. Further inventions included a high-frequency alternator, and an alternator for producing sine waves of 2,000 frequency.

Duddell's activities were not confined to research work. He made a reputation as a brilliant lecturer who was exceedingly happy in his choice of illustrative experiments. He was an active member of many scientific societies, and by his good judgment and tact was able to render valuable services to them. In 1907 he was President of the Röntgen Society. He passed through all the grades of membership of the Institution of Electrical Engineers, and was President from 1912 to 1914. He did invaluable work for the Institution as Chairman of the Library Committee, and the modern character of the Library is largely due to his efforts. He also keenly interested himself in the Institution Museum and in the Science Museum at South Kensington. He acted as Treasurer of the Physical Society of London from 1910-17. It was largely due to his efforts that the Physical Society was able to publish its Proceedings in their present form. He was elected a Fellow of the Royal Society in 1907, and was awarded the Hughes Medal in 1912.

During the war he devoted much time to the solution of scientific problems which the war brought into prominence, and it is probable that excessive work in this direction was responsible for his premature death. Duddell was always willing to help anyone who submitted an electrical problem to him. He brought to his work the skill of the trained mechanic, and himself constructed the most delicate parts of his first models. Over and above sound theoretical attainments, he possessed a wonderful

knowledge of materials and the methods of handling them, together with an extensive command of modern technical literature. R.S.Wh.

A strong desire having been widely expressed that the work of Duddell should be fitly commemorated, the Council of the Physical Society in February, 1920, appointed a committee to consider the proposal. The committee consisted of Sir William Bragg, Sir Horace Darwin, Sir Richard Glazebrook, Prof. T. Mather, and Mr. R. S. Whipple, with Dr. R. Knox (representing the Röntgen Society), and Mr. Roger T. Smith (representing the Institution of Electrical Engineers). As a result the Council in October, 1920, resolved to institute a Duddell Memorial Medal. A gratifying response was made to the appeal for funds, the sum of £639 5s. 8d. being subscribed by 213 individuals. The Physical Society made a contribution of £50, and the Röntgen Society of £10 10s. Thus by April, 1923, including accumulated interest, a sum of £739 18s. 4d. was at the disposal of the Committee. The medal was designed by Mrs. Mary Gillick. Mr. Emery Walker prepared a diploma to accompany the medal. The first award will be made in 1924.

Illustrations of the Medal accompany the present Part.

Proceedings at the Meeting held on May 11, 1923, at the Imperial College of Science.

The President, ALEXANDER RUSSELL, M.A., D.Sc., in the Chair.

THE DUDDELL MEDAL.

SIR WILLIAM BRAGG, F.R.S., Chairman of the Duddell Memorial Committee, handed over to the President of the Physical Society a copy of the Memorial Medal, together with the dies and a certificate for the unexpended balance of the memorial fund. In the course of his remarks Sir William Bragg said that the committee had been formed in response to a widely felt wish that the work of this distinguished scientist should be commemorated in a suitable manner. A sum of no less than £700 had been subscribed, thanks in the main to the energy of Mr. R. S. Whipple, secretary to the committee. As regards the design of the medal, great pains had been taken to ensure that it should be worthy of the memory of the dead and of the art of the nation to which he belonged. The work had been entrusted to Mrs. Mary Gillick, and the committee felt that their choice had been fully justified by the result. Mrs. Gillick had devoted immense trouble and care to her task, and the medal she had produced was one of which the Society might be proud.

Photographic slides of the medal were exhibited. On the obverse is a miniature of Mr. Duddell; and on the reverse a figure symbolical of Science examining the world as she finds it, with legends "THE PHYSICAL SOCIETY OF LONDON" and "RERUM NATURAM EXPANDERE."

DR. ALEXANDER RUSSELL, President, in accepting the medal on behalf of the Society, said that he did so with gratitude to all who had combined to make this memorial possible, and particularly to Sir William Bragg and Mr. R. S. Whipple, whose initiative had set the undertaking on foot, while their perseverance had carried it to such a successful issue. He had been associated with Mr. Duddell at the City and Guilds Technical College and in other connections. He had always

marvelled at the encyclopædic character of Mr. Duddell's technical knowledge, which, coupled with his mechanical skill and inventive ingenuity, had enabled him to make so many contributions of permanent importance to the progress of science and of industry. He had been Treasurer, Member of Council, and Vice-President of the Physical Society, and President of the Institution of Electrical Engineers.

SIR RICHARD GLAZEBROOK said that it gave him great pleasure to express his warm and intense admiration for Mr. Duddell. The latter had been a cordial helper of the National Physical Laboratory, especially during the war, and had always been ready to give generously from his great store of knowledge and of scientific resource. His memory would be kept alive not only by his own work, but by the admirable medal which had been designed; it was to be hoped that the latter would encourage others to follow his example and to engage in the field of scientific enterprise in which Duddell had been such a prominent worker.

TERMS OF AWARD OF THE DUDDELL MEDAL.

The following resolution was adopted by the Council on May 11, 1923 :—

DUDDELL MEMORIAL MEDAL.

Resolution.—That the £400 War Loan 5 per cent. 1929/47 Inscribed Stock be accepted in trust from the Duddell Memorial Fund Committee, and that the income therefrom be expended in accordance with the following regulation :—

A bronze medal shall be awarded by the Council not more frequently than once a year to persons who have contributed to the advancement of knowledge by the invention or design of scientific instruments or by the discovery of materials used in their construction. The award shall be made without restriction as to nationality or Fellowship of the Society. A parchment Certificate of Award and a sum of money may accompany the medal.

Should more than one person be connected with the invention or discovery for which the award is made, the Council may at its discretion present more than one medal and apportion the money accordingly.

The terms of award shall not be varied except by Special Resolution passed by the Society at a General Meeting and confirmed at a subsequent Meeting.

THE EIGHTH GUTHRIE LECTURE.

"*The Present Position of the Radiation Problem.*"

By J. H. Jeans, *D.Sc., LL.D., F.R.S.*

Summary.

Since about 1900 it has been obvious that classical dynamics conflicts with experience in certain respects, and particularly with reference to the radiation problem. According to the classical dynamics, for instance, (1) radiation in a steady state with matter should tend to run entirely into waves of very high frequency, and (2) the spectrum of any element, say hydrogen, should be continuous, whereas the observed spectrum is a line spectrum. These contradictions can indeed be shown to occur under any system of mechanical laws which is entirely continuous. The discrepancies with experience accordingly suggest that the laws of nature must in some way be discontinuous. To explain the observed nature of black-body radiation, Planck propounded the quantum-theory; in the hands of Bohr it soon became apparent that the quantum-theory contained also the clue to the line spectrum.

Hence arose hopes of rapid development which should explain the whole of the radiation problem and reduce molecular physics to order. The more sanguine of these hopes have not been fulfilled. Progress has been slow and difficult. No royal road has been discovered and what progress there has been is concerned with the structure of matter and not with radiation.

One solitary attempt at progress with the radiation problem must be mentioned, namely, Einstein's hypothesis of light quanta, which assumed light to be of an almost corpuscular nature, or at any rate to move in complete quanta. The obvious advantage of this hypothesis was that it gave an immediate explanation of the photo-electric effect; indeed, it enabled Einstein to predict the then unknown law connecting the electronic velocities with the frequency of the radiation. Incidentally it also provided a simple explanation of Planck's radiation formula. The hypothesis soon had to give way before the destructive criticism of Lorentz and others. In brief this criticism centred round the impossibility of reconciling the hypothesis with the known facts of the undulatory theory of light. A direct experimental test by G. I. Taylor also gave results which must be regarded as fatal to the hypothesis of light quanta.

Hence has become established a general belief that the classical theory as expressed by Maxwell's equations for free ether must remain true at distances sufficiently removed from matter. In the general Maxwell equations—

$$\frac{4\pi}{c}\rho u+\frac{1}{c}\frac{dX}{d}=\frac{\partial\gamma}{\partial y}-\frac{\partial\beta}{\partial z}$$

$$-\frac{1}{c}\frac{d\alpha}{dt}=\frac{\partial Z}{\partial y}-\frac{\partial Y}{\partial z}$$

it appears to be the first term ρu that requires modification. All the terms except this are concerned with the flow and spread of radiation in free space; this term is concerned with the interchange of energy between radiation and matter.

The different methods of interchange of energy between matter and ether or radiation may be divided into three classes which we may call sub-atomic, atomic and mass transfers. These three mechanisms of interchange must, of course, all give the same formula for black-body radiation, and we can test any hypothetical mechanism by examining whether it would by itself lead to Planck's formula.

Typical of a sum-atomic interchange is the emission or absorption of radiation by a Bohr atom. Here the equation covering a transfer of energy ΔW is

$$\Delta W = h\nu.$$

Einstein, in a very remarkable Paper which appeared in the Phys. Zeits., in 1917, showed that this law, subject to certain assumptions to be considered later, leads to Planck's law.

Typical of an atomic transfer of energy is the emission and absorption of heat by a solid. Debye's theory of specific heats indicates that here again the law of transfer is given by the same equation—

$$\Delta W = h\nu.$$

Typical of a mass transfer is the transmission of momentum occurring when a beam of radiation falls upon the surface of a perfect reflector moving with a known velocity u. Here it is easily shown that the transfer of energy cannot be in accordance with the above equation, since the transfer of energy ΔW can be made to have any value we please, positive or negative, by varying the frame of reference according to which the velocity u is measured. In this case, since the transfer of energy clearly cannot be by whole quanta, it is natural to suppose that it must be in accordance with classical mechanics. This does not lead, it is true, to Planck's law, but as is well known, it leads to Wien's law, which includes Planck's law as a special case. The effect of a transfer of energy of this type is not to diminish or increase the number of quanta of radiation, but merely to change the frequency, and so also the energy, of the quanta already in being.

We are in this way led simply and almost inevitably to a consistent view of the transfer of radiation—namely, that physical and chemical transfers take place by quanta while mechanical transfers take place according to the classical laws.

The application of these general principles enables us at least to conjecture what may happen in special problems. Consider, for instance, the exchange of energy between a free electron and a field of radiation. This interchange cannot be covered by the classical laws, since it is easily seen that these would lead to the equipartition formula $8\pi RT\lambda^{-4}d\lambda$ for black-body radiation and not to Planck's law. Further, the interchange of energy cannot be by quanta, for, if an electron absorbed a quantum $h\nu$ of energy, its final velocity v would be given by an equation of the type

$$\tfrac{1}{2}mv^2 = \tfrac{1}{2}mu^2 + h\nu.$$

The two terms on the right of this equation are necessarily positive, while the term on the left is zero for one observer—namely, for an observer moving with the final velocity of the electron. For this observer the equation makes zero equal to the sum of two positive quantities, which is an absurdity. Since a free electron can apparently absorb or emit neither according to the classical mechanics nor according to the quantum law, it seems probable that it can have no exchange of energy at all with a field of radiation. It is probably only in the case of bound electrons that interchanges of energy can take place.

In the limiting case in which the radiation is of zero frequency the electric forces approximate to those of an electrostatic field. In the extreme limit in which the field actually is electrostatic, a large mass of experimental evidence seems to suggest that electrons are acted on by the mechanical forces given by the classical formula. It seems probable that an electron e in a field of force X experiences a mechanical force eX when the field is steady, but not when it is a field of radiation. We may, perhaps, conjecture that the true force is given by a formula of some such general type as

$$\frac{e}{t}\int X dt,$$

but it has to be admitted that at present this cannot be anything more than conjecture. We may express the same idea somewhat differently by supposing the force X expanded in a Fourier series in the form

$$X = X_0 + \Sigma X_p \cos pt,$$

and supposing that the force X_0 acts on the electron in the manner supposed by the classical laws, while the forces represented by X_p do not influence the electron at all except through cataclysmic motions of the kind contemplated in Bohr's theory. We may perhaps regard the values of X_p, &c., as giving an indication of the probability of cataclysmic motion occurring. A conception of this kind is used by Einstein in his 1917 derivation of Planck's law, to which I have already referred. It seems difficult to interpret this conception of Einstein's except by supposing that the electric forces with which we are familiar are to be regarded as manifestations of a sub-universe more fine grained than anything we have so far imagined. Although some thought of this kind seems to be suggested by Einstein's Paper, most of us will feel that many possibilities must be explored before we find ourselves driven to believe in a sub-universe of this kind. On the other hand, the direct evidence of the nature of radioactive transformation gives considerable support to Einstein's hypothesis together with its accompanying suggestion of some sort of sub-universe.

The general conception that X_p, &c., instead of measuring mechanical forces on electrons, may give an indication as to the probability of the occurrence of cataclysmic motion, appears to give promise of removing many of the difficulties which now beset molecular physics and the theory of radiation. If $p/2\pi = \nu$, the probability of a cataclysmic exchange of energy of amount $h\nu$ may be supposed to depend only on X_p and ν; to secure agreement with the classical electrodynamics in the limiting case of $\nu = o$, we may assume the probability to be proportional to $X_p^2/h\nu$. If radiation, generated at a point P, spreads out in spherical waves in accordance with Maxwell's equations, the value of X_p^2 at Q will fall off as $1/PQ^2$, while ν will not vary with PQ at all, so that $X_p^2/h\nu$ will vary as $1/PQ^2$. In the special case in which monochromatic radiation of frequency ν is alone concerned, any transfer of energy to matter must be by quanta of amount $h\nu$ and the probability of such a transfer falls off as $1/r^2$. There is no longer any conservation of energy as between the radiation and the matter in any small region of space, but the conservation of energy now reappears as a statistical law, of validity somewhat similar to that of the second law of thermodynamics. There is no longer any contradiction between the theory of quanta and the undulatory theory of light, and such phenomena as the photoelectric effect appear as the inevitable consequence jointly of the quantum equations and of Maxwell's equations for free ether.

XXII. *The Effect of Torsion on the Thermal and Electrical Conductivities of Metals.*

By CHARLES H. LEES, *D.Sc., F.R.S., and* J. E. CALTHROP, *B.A., B.Sc.*

RECEIVED MAY 7, 1923.

ABSTRACT.

A method is described which enables the effect on the thermal conductivity of a wire of twisting the wire to be measured. In each of the steel, aluminium, copper and lead wires tested the twist decreases the conductivity along the wire by a small amount which is approximately proportional to the square of the twist per unit length.

The change of electrical conductivity is found to be in general less than the change of thermal conductivity, but it is also approximately proportional to the square of the twist per unit length.

I.

THE close connection which all forms of the electron theory of flow of heat and electricity in metals disclose between the conductivities k for heat and K for electricity makes it desirable that these quantities should be determined for as many metals and under as many different conditions as possible in order to see the extent to which the theories agree with the facts.

Both conductivities have been measured for a number of metals and alloys at temperatures between 18° C. and 100° C. by Jäger and Diesselhorst* and between −180° and 15° C. by Lees.† The latter observations show that the constancy of the quantity $\frac{k}{KT}$ where T is the absolute temperature which is maintained above 0° C. for the pure metals, ceases for them at lower temperatures, and both sets of observations show that the quantity has not the same value for alloys as for pure metals, nor does it remain constant as the temperature is varied.

Johnstone‡ has shown that when a wire of copper, zinc, aluminium, nickel, iron, steel or brass is stretched, the conductivity for heat along it is increased slightly (0·5 per cent.) by a tensile stress about 0·7 of the elastic limit for the material, while the measurements of Tomlinson, Gray, Smith, Credner and others§ show that the electrical conductivities of metals are under the same circumstances decreased by approximately 1/10 this amount.

Lussana‖ and Bridgman¶ have measured the effect of pressure on the heat conductivities of a number of metals, but unfortunately their results differ as to both sign and magnitude of the effect in many cases. Bridgman finds the conductivity increases with pressure for lead, tin and zinc, and decreases for iron, copper, silver, nickel, platinum, bismuth and antimony, at rates which vary greatly from one material to another.

* Jäger and Diesselhorst, Abh. Phys.-Tech. Reichsanstalt 3, p. 269 (1900).

† Lees, Trans. Roy. Soc., A. 208, p. 381 (1908).

‡ Johnstone, Proc. Phys. Soc. 29, p. 195 (1917).

§ *See* Lees, Proc. Phys. Soc. 29, p. 203 (1917), for a discussion of the question and references to further determinations.

‖ Lussana, Nuovo Cim. 15, p. 130 (1918).

¶ Bridgman, Proc. Amer. Acad. 57, p. 77 (1922).

By combining these results with his observations of the effect of pressure on the electrical conductivities* Bridgman shows in his Paper of 1922, referred to above, that the quantity $\frac{k}{KT}$, which should on the usual electron theories remain constant, changes very considerably under pressure, and he has put forward a modified electron theory of conduction† in better accord with the observed facts.

Since the appearance of Johnstone's Paper, Bridgman has given the results of his measurements of the effect of tension on the electrical conductivities of eight metals and alloys.‡ For strontium, bismuth, manganin and therlo the conductivity increases with increase of tension at rates of the order 10^{-6} of itself per kilogram per square cm. of tension, and for lithium, calcium, antimony and cobalt it decreases at rates of the same order.

The normal metal, according to Bridgman's measurements, decreases in electrical conductivity under tension, as was found by the earlier observers. In the case of the normal metals tension thus increases the thermal and decreases the electrical conductivity.

II.

So far as we are aware, the only determinations of the effects of torsion on the thermal conductivities of metals are those of Smith§ on rods of iron, steel, copper and brass $\frac{5}{16}$ in. diameter and 4·5 ft. long, which showed decreases proportional to the twist. Both Smith and von Szily|| have determined its effects in the electrical conductivities, and find decreases approximately proportional to the twist.

If the metal is obtainable in the form of wire the effect on the electrical conductivity can readily be determined by either the bridge or the potentiometer method. The measurement of the effect on the thermal conductivity is not so easily made to the accuracy necessary, but a method similar to that used by Johnstone for the determination of the effect of tension seemed suitable, and it has proved successful.

III. Outline of Method.

The material to be tested is in the form of a wire the ends of which are held in water-cooled clamps, one of which can be rotated through a measured angle with respect to the other to apply the twist. The heat is supplied to the centre of the length of wire by an insulated coil of manganin wire wound round the test wire, through which a measured electric current is sent. The temperatures at two points on the same side of the centre of the wire are measured by two insulated platinum wires wound on the wire under test. The platinum thermometers can be connected to a resistance bridge in such ways that either their resistances or the difference of their resistances can be measured.

The wire under test is entirely surrounded by a water jacket kept at the same temperature as the clamps. Measurements of resistances and differences are made with and without torsion, sufficient time being allowed between two sets of observa-

* Bridgman, Proc. Amer. Acad. 52, p. 573 (1917), 56, p. 61 (1921), and 58, p. 151 (1923).
† Bridgman, Phys. Rev. 19, p. 114 (1921).
‡ Bridgman, Proc. Amer. Acad. 57, p. 41 (1922).
§ N. F. Smith, Phys. Rev. 6, p. 429 (1909).
|| v. Szily, C.R. 128, p. 927 (1899). Math. u. Nat. Ber. Ungarn 16, p. 298 (1898).

tions to ensure that all temperatures have become steady. A platinum coil in contact with the clamp or wall of the enclosure gives the temperature of the air surrounding the wire.

IV. Details of the Apparatus.

The wire under test, WW', Fig. 1, No. 14, diameter 0·2 cm., was held at each end by a clamp CC', which formed part of a cylindrical brass box through which water was kept circulating. The box C' could be rotated about its axis with respect to the box C through an angle measured by the dial and pointer, and could be fixed by the clamp. A hollow cylindrical water jacket surrounded both wire and boxes.

The heating coil H consisted of 14 cm. of No. 40 manganin wire, silk-covered, wound directly on the wire tested and covering 0·45 cm. of that wire. The ends of the manganin wire projected 0·5 cm. from the coil, and to them were soldered lengths of No. 30 cotton-covered copper wire leads. The resistance of the coil was 4 ohms. The current through the coil was supplied by a small storage cell, and was read and kept constant to within 1 part in 2,000 by means of a shunted Weston milliammeter and adjustable resistance. The platinum thermometers consisted each of 35 cm. of No. 40 single silk-covered platinum wire wound directly on the wire under test and covering 1·05 cm. of that wire. Each was provided with copper

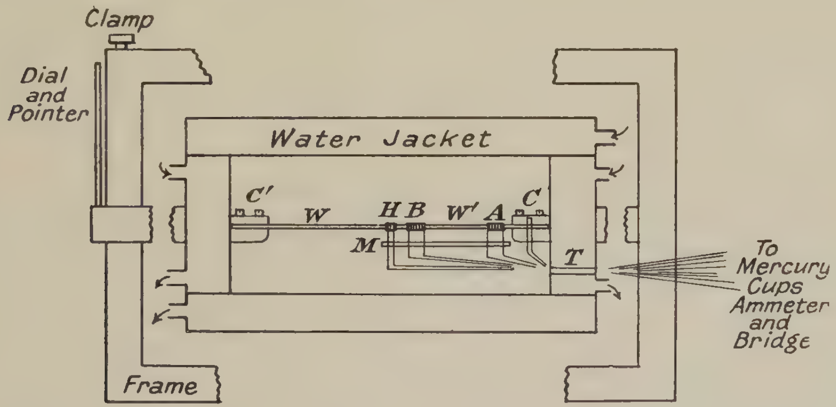


Fig. 1.—WW' Wire, H Heating Coil, A, B Thermometer Coils, M Mica Strip.

leads, as in the case of the heating coil. Their resistances were 2·8 ohms at 15° C. One B was placed near the heating coil and the other A near the clamp C. Their distances from each other and from the heating coil were maintained constant by means of a mica strip M, through which the leads were threaded. The diameter outside the windings of heating and thermometer coils was 0·25 cm. The thermometer giving the temperature of the clamps and surrounding jacket was similar in construction. The leads from the various coils emerged from the surrounding jacket through a tube T in the end cylinder C.

The resistances were measured by means of a Griffiths-Callendar bridge and Gambrell galvanometer, a deflection of 1 mm. on the galvanometer scale corresponding to a difference of resistance of 0·0002 ohms.

V. Theory of the Method.

If a wire within an enclosure at constant temperature has its ends secured to the enclosure so that they have the same temperature as the enclosure, while heat is supplied at a constant rate to the middle portion of the wire, the temperature of the middle portion will be raised above that of the enclosure and heat will flow along the wire in both directions from the middle to the ends. If Q is the quantity of heat which leaves the short heated portion of the wire in each direction per second, and v is the excess of the temperature at a distance x from the end of the wire over the temperature of the enclosure, we have*

$$v=\frac{Q}{qka}\cdot\frac{\sinh ax}{\cosh ax_0}, \quad \ldots \quad (5.1)$$

where q is the area, p the perimeter of the section of the wire, k the thermal conductivity of the material of the wire, h the emissivity of its surface, $a^2=ph/qk$, and x_0 is the distance of the near end of the heated portion from the clamp at the end of the wire.

If heat is supplied at a uniform rate to the surface of the wire between x_0 and the centre x_C, the mean temperature excess of the surface of the heated portion may be taken at that given by equation (5.1) for a point x_D one third of the distance between the edge x_0 and the centre x_C of the heated portion of the surface—that is for $x_D=x_0+\frac{1}{3}(x_C-x_0)$. If the total supply of heat per second to each half of the wire is H, then

$$H=Q+hsv_D=Q\left(1+\frac{hs}{qka}\cdot\frac{\sinh ax_D}{\cosh ax_0}\right) \quad \ldots \quad (5.2)$$

where s is the surface and v_D the mean temperature as thus determined.

If the mean temperature excesses v_A and v_B of two short lengths $2b$ of the wire whose centres are at distances x_A and x_B from the clamp at one end of the wire are observed, we have

$$Q=qka\cosh ax_0\left(\frac{v_B-v_A}{\sinh ax_B-\sinh ax_A}\right)\cdot\frac{ab}{\sinh ab}. \quad \ldots \quad (5.3)$$

Hence

$$H=(qka\cosh ax_0+sh\sinh ax_D)\frac{v_B-v_A}{\sinh ax_B-\sinh ax_A}\cdot\frac{ab}{\sinh ab} \quad \ldots \quad (5.4)$$

If S is the resistance of that portion of the heating coil encircling the wire† and A is the current through it in amperes,

$$H=SA^2/4.18 \quad \ldots \quad (5.5)$$

If the resistances of the platinum thermometers at the points B and A when both are at the temperature of the water jacket (about 15°C.) are R_0 and r_0 respectively, and when heat is flowing along the test wire are R_1 and r_1 respectively then if γ is the temperature coefficient of the increase of resistance of the platinum wire,

* Lees, Phil. Trans. Roy. Soc., A 208, p. 381 (1908).

† Taken throughout as 13/14 of the whole heating coil wire.

i.e., $R_v = R_0(1+\gamma v)$*, and we take into account the fact that only 34 cm. of the 35 cm. of platinum wire is wrapped round the test wire, we find that

$$v_B - v_A = \frac{35}{34} \cdot \frac{1}{\gamma}\left(\frac{R_1}{R_0} - \frac{r_1}{r_0}\right) \quad \text{(5.6)}$$

On substituting these values in equation (5.4) we get

$$\left(qk + sh \,.\, x_D \frac{\sinh ax_D}{ax_D} \cdot \frac{1}{\cosh ax_0}\right) \frac{\cosh ax_0}{x_B \frac{\sinh ax_B}{ax_B} - x_A \frac{\sinh ax_A}{ax_A}} \cdot \frac{1}{\frac{\sinh ab}{ab}} = \frac{SA^2}{\frac{4 \cdot 34}{\gamma}\left(\frac{R_1}{R_0} - \frac{r_1}{r_0}\right)} \dagger \quad \text{(5.7)}$$

When k is fairly large as in the case of copper, a is small, the terms cosh x and sinh x/x approximate to unity, and may be expanded in ascending series. If terms above x^3 are neglected $\sqrt{k}$ disappears from the equation which becomes linear in k, and may be solved directly. When k is not large enough to make terms beyond x^3 negligible, the equation is best solved by successive approximation. With an assumed value of k taken from physical tables and with h taken 0·00030, a is found, and the value of the left-hand side of the equation calculated. On comparison with the value of the right-hand side it is seen whether the assumed value of k is too small or too large, and a new value is assumed accordingly. The calculation of the left-hand side is repeated, and the two values of it found when compared with the right-hand side allow a fairly close approximation to the true value of k to be found. The approximation is carried out to 1 part in 1,000. When the small change of value of k due to twist is to be calculated the process is carried out easily and quickly.

VI. Methods of Observation.

Water from the mains was circulated through the space between the double walls of the enclosure and electric current through the heating coil for about an hour in order to allow the steady state to be attained. The resistance of the enclosure coil was then read on the Callendar-Griffith bridge, then that of the hotter coil, and finally the difference between the hotter and the cooler coil. The heating current was kept constant. The current was then stopped. About 10 minutes was necessary for the wire to take up throughout its length the temperature of the enclosure. Resistance observations were now made in the order difference, hotter, enclosure. Unless the final temperature of the enclosure agreed closely with the initial temperature the experiment was regarded as unsatisfactory and was repeated.

A twist of 5—10^0 was usually given to one end of the wire and the observations repeated. The twist was then increased and further observations made. In some cases a still greater twist was applied. Finally the twist was removed and the observations repeated.

The positions of the heating and thermometer coils were determined before and after the observations to ensure that no change had taken place.

* It may be noted that γ is not the coefficient of increase of resistance of platinum at 0°, that is 0·00392, but 0·00392/(1+15×0·00392), that is 0·00368.

† In this term a slight correction has been made as the thermometer coils are approximately 1 per cent. cooler than the wire with which they are in contact, owing to the electrical insulation of the platinum wire.

The following observations taken with a copper wire show the accuracy and consistency of the readings :—

No. 14 copper wire for electrical use, diameter 0·20 cm., length between clamps 13·60 cm. $x_A=1{\cdot}37$ cm., $x_B=5{\cdot}87$ cm. Length of each thermometer coil 1·05 cm., of heating 0·40 cm. along wire. Date, 1923/11/2. Water bath at start 14·63°C., at end 14·67°C.

Twist per cm.	Heating.	Thermometer readings, ohms.			
		Case before.	A.	B.	Case after.
0	On.	2·8966	3·0066	3·0042	
0	Off.		2·8657	2·8648	2·9015
0·51 *L*	On.	2·8978	2·9050	3·0044	
0·51 *L*	Off.		2·8624	2·8619	2·8990
0	On.	2·8965	2·9041	3·0088	
0	Off.		2·8621	2·8612	2·8970

The fourth figure after the decimal point represents hundredths of a degree C. very nearly.

The electrical conductivity was measured by the potentiometer method, a constant current being sent through the wire and the fall of potential down a given length determined with the wire twisted and without twist. In some cases the resistance of a considerable length of the same wire was measured directly by the bridge method and the change on twisting found.

VII. Tables of Results.

(1) Steel wire 0·203 cm. diameter, 13·6 cm. long of mild steel, from the London Electric Wire Co.

Heat Conductivity.

Distance between temperature coils 4·5 cm. Heating current 0·200 amps. *k* by equation (5·7), 0·19.

Date.	Twist. (Deg. per cm.)	(Twist)^2.	Conductivity.	Per cent. decrease.
1922/7/18	0	0	1·0	0
	0·22 *Left*	0·048 *L*	0·9975	0·25
	0·51 *L*	0·26 *L*	0·9895	1·05
	0·74 *L*	0·55 *L*	0·9850	1·5
	0·88 *L*	0·77 *L*	0·9785	2·15
/7/19	0	0	0·9925	0·75
/7/18	0	0	1·0	0
	0·88 *Right*	0·77 *R*	0·9980	0·2
	1·48 *R*	2·19 *R*	0·9870	1·3
	0	0	0·9955	0·45
/7/12	0	0	0·9980	0·2
Electrical Conductivity.				
1922/10/10	0		1·0	
	0·74 *R*	0·55 *R*	0·999	0·1
	1·84 *R*	3·4 *R*	0·993	0·7
	0	0	0·996	0·4
/10/11	0	0	1·0	
	1·10 *L*	1·2 *L*	0·998	0·2
	1·47 *L*	2·2 *L*	0·996	0·4
	1·84 *L*	3·4 *L*	0·993	0·7
	0	0	0·998	0·2

(2) Aluminium wire 0·203 cm. diameter, 13·6 cm. long, of commercially pure metal, from the London Electric Wire Co.

Heat Conductivity.

Distance between temperature coils 4·6 cm. Heating current 0·300 amperes. k by equation (5·7), 0·39.

Date.	Twist.	(Twist)².	Conductivity.	Per cent. decrease.
1922/12/18	0	0	1·0	0
/19	1·47 *L*	2·2 *L*	0·9980	0·2
/20	0	0	1·000	
/21	1·47 *R*	2·2 *R*	0·9980	0·2
	1·84 *R*	3·4 *R*	0·9910	0·9
1923/1/15	0		1·0	
1923/1/15	0		1·0	
	1·47 *L*	2·2 *L*	0·9975	0·25
	1·84 *L*	3·4 *L*	0·9930	0·7
1/17	0	0	0·9985	0·15

Electrical Conductivity.

1923/2/14	0	0	1·0	
	1·10 *R*	1·2 *R*	0·9997	0·03
	1·36 *R*	1·85 *R*	0·9994	0·06
	1·84 *R*	3·4 *R*	0·9984	0·16
	0	0	1·0	0
	1·36 *L*	1·85 *RL*	0·9976	0·24
	1·84 *L*	3·4 *L*	0·9965	0·35
	0		1·0	

(3) Copper wire of 0·203 cm. diameter, 13·6 cm. long, of high conductivity copper, from the London Electric Wire Co.

Heat Conductivity.

Distance between temperature coils 4·5 cm. Heating current 0·410 ampere. k by equation (5·7), 0·99.

Date.	Twist.	(Twist)².	Conductivity.	Per cent. decrease.
1922/10/19	0	0	1·0	
	0·59 *R*	0·35 *R*	0·9990	0·1
	0	0	0·9990	0·1
/10/20	0	0	1·0	
	0·74 *R*	0·55 *R*	0·9965	0·35
	0	0	0·9965	0·35
/11/1	0	0	1·0	
	0·65 *L*	0·42 *L*	0·9955	0·45
	0	0	1·0	
/11/2	0	0	1·0	
	0·51 *L*	0·26 *L*	0·9975	0·3
	0	0	0·9999	0·01

Electrical Conductivity.

	0	0	1·0	
	0·74 *L*	0·55 *L*	0·9985	0·15
	0	0	1·0	0

(4) Lead wire 0·195 cm. diameter, 13·6 cm. long, of commercially pure lead, from Messrs. Johnson & Matthey.

Heat Conductivity.

Distance between temperature coils 4·50 cm. Heating current 0·24 ampere. k by equation (5·7), 0·092.

Date.	Twist.	(Twist)².	Conductivity.	Per cent. decrease.
1923/1/30	0	0	1·0	
	0·29 *R*	0·08 *R*	1·0	0
	0·59 *R*	0·35 *R*	0·9965	0·35
	0	0	1·0	0
/2/5	0	0	1·0	
	0·81 *L*	0·66 *L*	1·0	0·0
	1·10 *L*	1·2 *L*	0·9975	0·25
	0	0	1·0	0
/2/6	0	0	1·0	
	0·59 *L*	0·35 *L*	0·9980	0·2
	0	0	1·0	0
/2/8	0	0	1·0	
	0·90 *L*	0·8 *L*	0·9965	0·35
	0	0	1·0	0

Electrical Conductivity.

	0	0	1·0	
	0·90 *R*	0·8 *R*	0·9985	0·15
	0	0	1·0	0

Fig. 2 shows graphically the results obtained.

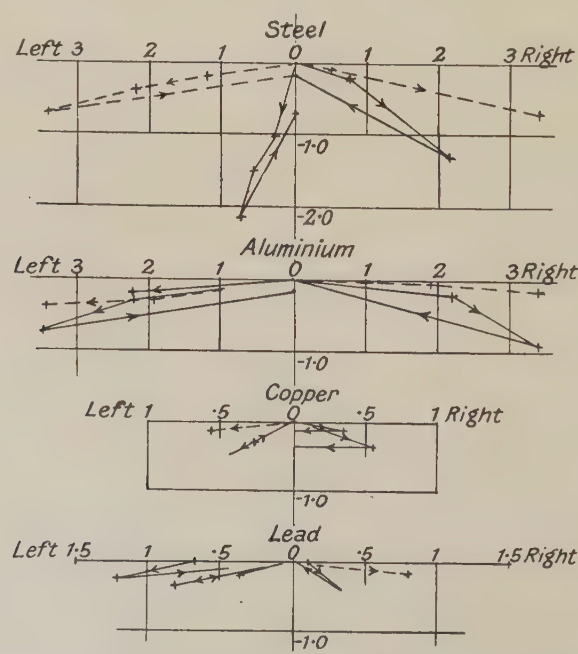


Fig. 2.—Abscissae represent Twist in Degrees per cm.², Ordinates per cent. Decrease of Conductivity, Full Line Thermal, Dotted Electrical.

VIII. Discussion of the Results.

It will be noticed that in several cases the heat conductivity of a wire after the wire has been twisted does not return immediately to the value it had before the twist. This is so especially in the case of steel. The cause of this we have not yet been able to trace, but the same difficulty was encountered by Smith apparently to a greater degree. Apart from this irregularity the effect of twist in most of the wires tested is to decrease the heat conductivity by a few parts in in a thousand. In steel only does the decrease exceed 1 per cent. The decrease appears to be independent of the direction of twist and for small twists proportional to the square of the twist. Its magnitude agrees as well as could be expected with that found by Smith, who took it as proportional to the twist.

The effect of twist on the electrical conductivity is in all cases less than that on the heat conductivity, but follows roughly the same law—that is, it is proportional to the square of the twist. Its magnitude is of the same order as those found previously by Kelvin, Witkowski, Ascoli and von Szily,* who took it as proportional to the twist.

DISCUSSION.

Mr. C. R. Darling said that the method described seemed an admirable one, but wished to know whether the wires were annealed before the experiment? The initial condition of the specimens would doubtless affect the results obtained, and might account for the discrepancies between the conclusions arrived at by different observers.

Mr. Rollo Appleyard also referred to the effect of annealing the wire under test, and added that the effect of stretching on resistance is at least as important as that of twisting. In stranding copper wires for a cable the outer wires are stretched as well as twisted, and any permanent increase of resistance due to such causes is of some consequence in practice. It would be interesting if the experiments could be carried out on a large scale, instead of on a laboratory scale.

Prof. A. O. Rankine inquired whether the effect differs according as the direction of twist is clockwise or counter-clockwise as viewed in the direction of flow of the heat or electric current.

Mr. F. E. Smith suggested that the experiments might be simplified by means of an arrangement for producing a potential difference between the two clamps C,C' (Fig. 1 of the Paper) at the ends of the wire. The necessary heating would be effected by the current thereby set up along the wire, which would have a maximum temperature at its middle point owing to the cooling effect of the masses of metal at its ends. The required thermal and electrical measurements could be made by means of wires twisted on to the principal wire as described in the Paper.

Dr. Alexander Russell expressed his admiration for the methods of measurement devised by the authors. He pointed out that the twisting of the strands in a stranded cable produces a somewhat similar effect. For example in a seven-strand cable the centre strand is straight and the other six strands form helices enclosing it. If r be the resistance of the central strand the resistance of the cable is greater than $r/7$, for the bulk of the current continues to flow along the helical strands and little of it flows from strand to strand. Hence, if the "lay" of the helical strands be 1 in n (*i.e.*, if the pitch of a helical strand be n times its diameter) then the resistance R of the cable is given by

$$1/R=1/r+6\Big/\left\{r\left(1+\frac{\pi^2}{n^2}\right)^{\frac{1}{2}}\right\},$$

and thus

$$1/R=(7/r)(1-3\pi^2/7n^2)$$

* Kelvin, Nature, 18, p. 180 (1878). Witkowski, Trans. R. Soc. Edin., 30, p. 413 (1881), and Ann. der Phy., 16, p. 161 (1882). Ascoli, Mimori. Acc. Lincei, 4, p. 406 (1887), Rend. Acad. Lincei, 6, p. 502 (1890), and 1, p. 10 (1892). Von Szily, Journ. de Physique, 8, p. 329 (1899), found the electrical resistance of constantan increased proportionally to the twist up to the elastic limit. For a constant twist the resistance decreased slowly with time. Less accurate observations on copper, nickeline and maillechort showed the same general results.

approximately when $(\pi/n)^2$ is small. Hence, the diminution of the conductance of the cable due to the lay of the wires is proportional to the square of the twist.

Prof. LEES, in reply to the discussion, said that the wires were soft and not annealed. The object of the experiments was to test the electron theory of conduction, and it was sufficient for this purpose that the state of the wire should merely be the same for the electrical as for the thermal measurements. The large scale tests which had been suggested presented attractions for the engineer, but any accuracy lost by performing the experiments on a small scale was fully compensated by the more precise methods available in a laboratory, while the relatively small cost of the latter methods is a serious consideration to the physicist. The effect of stretching has been investigated by Johnstone, and a reference to his work is given in the Paper. The thermal conductivity is increased by stretching while the electrical conductivity is diminished—another instance of failure in the relation predicted by the electron theory. Prof. Rankine's suggestion was an interesting one. It had been considered, but rejected as improbable; it might, however, be worth investigation. The method suggested by Mr. Smith was somewhat similar to that of Jäger and Diesselhorst, but the latter requires the thermal flow to pass through joints, and inaccuracies arise from this circumstance. As regards stranded cables, the effects of twisting and stretching are in opposite directions for thermal conductivity, but aid one another in the case of electrical conductivity, so that the matter would probably repay investigation.

XXIII. *The Use of the Wien Bridge for the Measurement of the Losses in Dielectrics at High Voltages, with Special Reference to Electric Cables.* By A. Rosen, A.C.G.I., B.Sc. (Engineering), A.M.I.E.E.

Received March 25, 1923.

ABSTRACT.

In the preliminary section, the loss angle of an imperfect condenser is defined, the equations for the Wien bridge are derived, and the effects of variations of frequency and voltage on the balance are discussed.

One of the difficulties in the application of large potential differences to a bridge is the effect on the arm which has to withstand the high voltage. In the arrangements due to Monasch and Schering, this arm is the known condenser; in the bridge as used by the author, the voltage is applied to the ratio coils. The errors introduced by earth impedance are eliminated by using the Wagner auxiliary bridge.

The applications to measurements on cables are considered, and the use of the double bridge in determining the "wire-to-wire" and "wire-to-sheath" losses in a multi-core cable is described.

In Appendix I. the corrections due to imperfections of the bridge arms are discussed, and in Appendix II. a simple quantitative theory of the double bridge is given.

Preliminary.

Loss Angle of a Condenser.

Consider a periodic E.M.F. of pure sine form and frequency $\frac{\omega}{2\pi}$, represented by $v=\hat{V}\sin\omega t$, applied to the terminals of a condenser; the resulting current, which may possibly be complex, can be represented in the general case by

$$i=\hat{I}\sin(\omega t-\varphi)+\hat{I}_3\sin(3\omega t-\varphi_3)+\hat{I}_5\sin(5\omega t-\varphi_5)+\ .\ .\ .\ .$$

The mean power dissipated will be $W=\frac{1}{T}\int_0^T vidt$, T being taken over a whole number of periods, $=\frac{1}{2}\hat{V}\hat{I}\cos\varphi$, so that, in measuring the power, we may disregard the harmonics in the current wave, and consider only the fundamental, $\hat{I}\sin(\omega t-\varphi)$. For any particular values of $\hat{V}$ and ω we can imagine the condenser replaced by a combination of a perfect, unvarying capacity C, shunted by a resistance which obeys Ohm's law, of conductance G, such that the total current is $i=\hat{I}\sin(\omega t-\varphi)$; C will be the effective capacity of our condenser, and the loss at a root-mean-square voltage V will be V^2G. The current leads on the P.D. by an angle $\varphi=\frac{\pi}{2}-\theta$, such that $\tan\theta=\frac{G}{\omega C}$, and the power may be expressed by $W=V^2\omega C\tan\theta$; θ is therefore a measure of the loss in the condenser, and is referred to as the "loss angle." The "power factor" as defined by $\int_0^T vidt\Big/\left(\int_0^T i^2dt\ .\int_0^T v^2dt\right)^{\frac{1}{2}}$ equals $\cos\varphi\ \frac{\hat{I}}{(\hat{I}^2+\hat{I}_3^2+\hat{I}_5^2+\ .\ .)^{\frac{1}{2}}}$, so that only when the current wave is free from harmonics can $\cos\varphi$ be strictly termed the power factor of the condenser; the difference can, however, usually be neglected.

The Wien Bridge.

The Wien Bridge,* a particular application of which is described in this Paper, measures the values of C and G, and thus θ directly. Diagrammatically it is represented in Fig. 1. The arms AB, AD consist of pure resistances R_1, R_2 respectively.

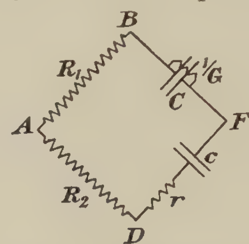


FIG. 1.—WIEN BRIDGE.

The unknown condenser to be measured is placed in the arm BF, and is represented by the capacity C shunted by the resistance $\frac{1}{G}$. The fourth arm is occupied by a perfect condenser c in series with a pure resistance r. An alternating P.D. of frequency $\frac{\omega}{2\pi}$ periods per second, and a suitable detector are placed across the conjugate points BD, AF. The impedance of the arm BF is $Z_3=\frac{1}{G+j\omega C}$, and of DF is $Z_4=r-\frac{j}{\omega c}$. The condition for no current in the detector is

$$\frac{R_1}{R_2}=\frac{Z_3}{Z_4}=\frac{1}{(G+j\omega C)\left(r-\frac{j}{\omega c}\right)}.$$

Equating the real and imaginary terms, we obtain, after transformation,

$$G=\frac{R_2}{R_1}\cdot\frac{\omega^2c^2r}{1+\omega^2c^2r^2}$$

$$C=\frac{R_2}{R_1}\cdot\frac{c}{1+\omega^2c^2r^2}$$

$$\tan\theta=\frac{G}{\omega C}=\omega cr.$$

In the majority of cases, $\tan\theta$ is a small quantity, and its square can be neglected in comparison with unity. Writing $\frac{R_1}{R_2}=m$, the expressions reduce to

$$G=\frac{\omega^2c^2r}{m}\ \text{mhos}$$

$$C=\frac{c}{m}\ \text{farads}$$

$$\theta=\omega cr\ \text{radians},$$

c and r being in farads and ohms respectively.

* M. Wien, Wiedemann's Annalen, Vol. 44, p. 689 (1891).

For a given set of conditions, G and C are fixed, and to satisfy these equations we must vary any two of c, r and m. In the arrangement described in detail below m is kept fixed, and c and r are varied for a balance.

Variation of Balance with Frequency.

This will depend on the properties of the dielectric being tested. It has been found experimentally* that for practically all the materials used as dielectrics for condensers the variation of the conductance G with frequency may be represented approximately by an equation of the form $G=a+b\omega$, where a and b are constants which are determined by the temperature ; also the capacity C is nearly independent of frequency. Substituting in the above equations, we obtain

$$r \propto \frac{a}{\omega^2}+\frac{b}{\omega}$$

$$c \doteqdot \text{constant.}$$

We may say that r varies inversely as some power of the frequency between the first and second, the precise number depending on the relative values of a and b. (For

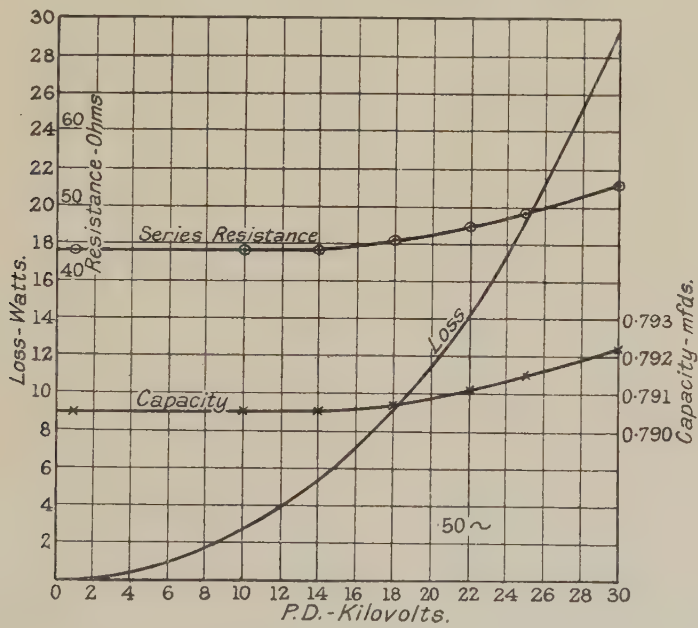


FIG. 2.—VARIATION OF BALANCE WITH VOLTAGE.

electric cables with impregnated paper insulation, at normal temperatures, a is relatively small, and r varies very nearly inversely as the frequency.)

It follows that if the current in the condenser tested is complex—i.e., contains terms of more than one frequency, the current in the detector can never be zero. However, by using an instrument that will respond to only one periodicity—e.g.,

* Fleming and Dyke, Journal I.E.E., Vol. 49, p. 323 (1912).

a tuned vibration galvanometer—a balance can be obtained for the fundamental, and the effect of the harmonics is eliminated. As no loss is caused by the harmonics, the result still gives the power spent in the dielectric. Further, if the P.D. applied to the bridge is not of pure sine form, the balance obtained will be in terms of the fundamental components of the voltage and current—i.e., the bridge will measure only the loss caused by the fundamental; in other words, the readings are independent of the wave-form of the applied voltage; this has been experimentally verified by Monasch.*

Variation of Balance with Voltage.

The change of capacity and loss angle of a condenser with voltage, if any, is generally not rapid, and, in any case, if the loss varies as V^n, to balance the bridge, r will vary approximately as V^{n-2}. This is a considerable advantage compared with the wattmeter method of measuring power, in which the reading varies as V^n, as firstly, the balance is not much disturbed by fluctuations of voltage, and, secondly, errors in the voltmeter which measures the P.D. are of less importance. In Fig. 2 are plotted some figures relating to a paper-insulated cable, and show the slow variation of r and c with voltage and the comparatively rapid change in the power loss.

PREVIOUS WORK.

Monasch's Method.†

The application of high voltages to a bridge presents certain difficulties which were only partially overcome by Monasch's arrangement. Referring to Fig. 3(a),

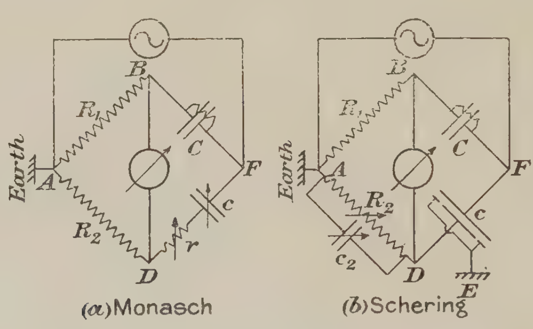


FIG. 3.—BRIDGES USED BY MONASCH AND SCHERING.

the P.D. was applied to the points AF, A being earthed, and a tuned optical telephone was placed as detector across BD. It will be seen that the full voltage on the condenser tested was applied to the adjustable capacity c, and this introduced serious limitations. After many efforts, a variable air condenser was constructed capable of withstanding 11 kilovolts without appreciable loss due to ionisation of the air, its maximum capacity being 230×10^{-6} mfd.; this set a definite upper limit to the testing pressure. The large surface area of this condenser introduced considerable earth capacity effect, so that the readings obtained were only

* B. Monasch, Electrician, Vol. 59, p. 416 (1907).

† Electrician, loc. cit.

comparative and not suitable for calculating the true loss angle. For his quantitative measurements, Monasch used a much lower P.D. and larger capacities, as he had concluded that the balance was independent of the voltage. As regards cable, this is only true for the comparatively low dielectric stresses used by Monasch and for the higher values it is necessary to test at the full voltage.

Schering's Method.

A more successful arrangement due to Prof. Schering, and described by Alfred Semm,* is shown in Fig. 3(*b*). It differs from Monasch's scheme in that the leakance in condenser C is compensated by a capacity c_2 shunted across R_2. The equations for balance are of the same form as previously, viz.,

$$C=\frac{R_2}{R_1}\cdot\frac{c}{1+\omega^2c_2^2R_2^2}$$

$$\tan\theta=\omega c_2R_2$$

and balance was obtained by varying R_2 and c_2. The air condenser c, which was of fixed capacity, had a "guard plate" connected to earth; this placed the "edge-effect" with the possibility of ionisation outside the bridge arm. The effect of the earth capacity of the L.T. plate was negligible, as it was in parallel with c_2, which was of the order of 1 mfd. By making the distance between the plates large, c was made capable of withstanding 100 KV., but the capacity was small, being only 50 cm., i.e., $55{\cdot}6\times10^{-6}$ mfd. A defect is that the impedances of the bridge arms were very unequal, causing loss of sensitivity; e.g., R_1 is given as usually 200 ohms, which is $\frac{1}{300,000}$ of the impedance of c at 50 cycles per second. Further, a serious practical disadvantage when testing cables is that the low-tension side of C has to be well insulated from earth, as any earth impedance shunts the arm R_1.

AUTHOR'S ARRANGEMENT.

The method used is shown in Fig. 4; it consists essentially of the usual Wien

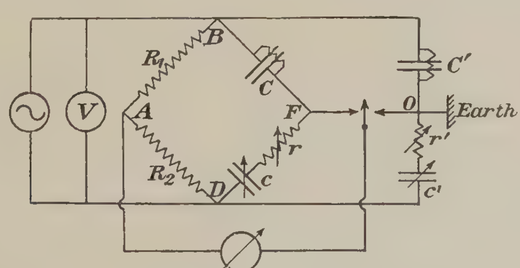


FIG. 4.—WAGNER DOUBLE BRIDGE.

bridge with the Wagner earthing arrangement.† The arms of the bridge *ABFD* are as in Fig. 1, and in addition an auxiliary bridge, consisting of the condensers C', c' and a resistance r', is placed across the points *BD*, the junction *O* of C' and

* Archiv für Elektrotechnik, Band 9, p. 29 (1920).
† K. W. Wagner, E.T.Z. Vol. 40, p. 1001 (1911).

r' being put to earth. The detector is switched alternately between the points AF, AO, c, r and c', r' being varied respectively to obtain a balance. When finally there is no deflection in either position, A, F and O are at the same, i.e., earth potential, and c, r give the true readings required. Considering the currents to earth from the points $ABFD$, those from B and D will merely modify the impedance of the arms BO, DO, and since A, F and O are at the same potential, no earth currents can flow from A and F. The effect of the impedance to earth of the apparatus, including the source and the detector, is thus eliminated.* (*See* Appendix II.)

The following points are to be noticed :—

(1) By connecting the source across the ratio coils, the arm of the bridge which has to withstand the high voltage is a resistance and not a capacity ; it is easier to construct a satisfactory resistance for this purpose than a condenser.

(2) By using a large value for the ratio $\frac{R_1}{R_2}$, e.g., 100 : 1, the P.D. across the measuring condenser c can be brought to within everyday values, so that ordinary condensers can be used for this purpose, and there is no limit as to their size.

(3) The galvanometer is brought to earth potential, and the P.D. between the adjustable arm FD and earth is small, so that the apparatus can be manipulated conveniently and in safety, which contributes to speed and accuracy in working.

(4) The voltage on the condenser tested is a fixed proportion of the P.D. of the source, which can be read directly on a voltmeter.

Practical Details.

Ratio Arms.

In obtaining the equations for the Wien bridge, it was assumed, for simplicity, that the phase angle of the resistances R_1, R_2 was zero. The equations still hold good if the resistances are not perfectly non-reactive, providing they have the same time-constant, in which case their ratio, m, has zero angle. This result can be largely attained by constructing both arms of identical units. R_1, R_2 consisted of 100 and one 5,000 ohm coils respectively, the total resistance being 505,000 ohms. They were wound on flat ebonite cards 0·036 in. thick with 0·002 in. d.s.c. Eureka wire in one layer, the direction of winding being reversed three times ; this form of winding gives a coil with small inductance and self-capacity suitable for withstanding high voltages. The cards were mounted in supports in such a way that the voltage and capacity between adjacent ones was small, and the whole was immersed in oil in a wooden tank supported on insulators. This resistance was capable of withstanding 30 KV. without undue temperature rise, and up to 50 KV. for a short time. A smaller coil of 2,500 ohms resistance was provided for use above 30 KV., giving a ratio of 200 : 1, and thus a lower voltage on the adjustable condensers.

For higher potential differences resistances become too elaborate, besides absorbing considerable power. In this case it is better to use inductances of the

* S. Butterworth, Proc. Phys. Soc., Vol. 34, p. 8 (1921).

same phase angle for the ratio arms. A tapping from the H.T. winding of the transformer would serve, but, apart from the difficulty of adapting an existing transformer, this has the disadvantage that the time-constant ratio would probably alter with the voltage and load. A more suitable arrangement would be a specially designed iron-cored choke coil, the winding being built up of a number of similar units in series. Each one of these would then have the same phase angle under all conditions, and wherever the tapping is made the two parts would have equal time constants.

Adjustable Capacity Arms.

For general work, three condensers were used in parallel as the variable capacity c, (1) mica dielectric, adjustable by means of plugs up to 20 mfds. in steps of 1 mfd., (2) mica dielectric of 3-decade type with dial switches giving continuous variation up to 1 mfd. in steps of 0·001 mfd., (3) air dielectric, continuously variable up to 0·0018 mfd. For the "extra" bridge capacity, accuracy of calibration and small loss angle are not needed, and Mansbridge condensers with paper dielectric were used for the values from 1 to 10 mfds.; a 3-decade condenser in parallel gave continuous adjustment from 0·001 to 1 mfd. The balancing was much facilitated by using the condensers with rotating switches.

Source.

The source of current used was an alternator of 60 KW. capacity directly driven by a D.C. shunt motor. The frequency could be varied by altering the excitation of the motor field, and was measured by a frequency meter of the vibrating-reed type, actuated by a device mounted on the main shaft. The meter could be read to within $\frac{1}{2}$ cycle per second, and a variation of about $\frac{1}{10}$ cycle per second could be detected.

Leads.

The lay-out of the testing plant was such that all parts at a dangerous potential were completely protected, e.g., the transformer was housed in an enclosed chamber, and the voltmeter guarded by a large glass screen. It was desirable that the addition of the apparatus described should not alter this general rule. The ratio arms, which have to withstand the full voltage, were therefore placed in the chamber alongside the transformer, and the galvanometer, variable condensers and resistances, which are all at low working potential, were placed conveniently near the controls for the frequency and voltage. This necessitated somewhat long connecting leads; these were brass-taped over the insulation and then laid up in pairs, the screens being joined to earth. Thus, the possibility of inductive interference from the primary circuit was avoided; further, the impedance between the leads was eliminated, as it was taken account of by the Wagner auxiliary bridge. This impedance consists, not only of the capacity and leakance of the leads, but also leakage over the ends, and this last may, in certain cases, cause serious errors unless avoided by having the leads screened.

When tests were taken on drums of cable, the P.D. was conveyed through highly-insulated leads, about 20 yards in length, consisting of two single core cables

with impregnated paper insulation, the lead sheathing being earthed. Here, again, the double bridge arrangement eliminated the impedance to earth of these leads, and no correction was needed to take account of them. The resistance of these H.T. leads was negligible, as the equivalent series resistance of the cable never amounted to less than several hundreds of ohms. However, the resistance of the leads to the adjustable arm *FD* could not in certain cases be neglected, and its value, 0·20 ohm., was added to the readings of the series resistance *r*.

Detector.

A vibration galvanometer made by H. Tinsley & Co. was used as detector A step-down transformer was found greatly to increase the sensitivity. (Actually

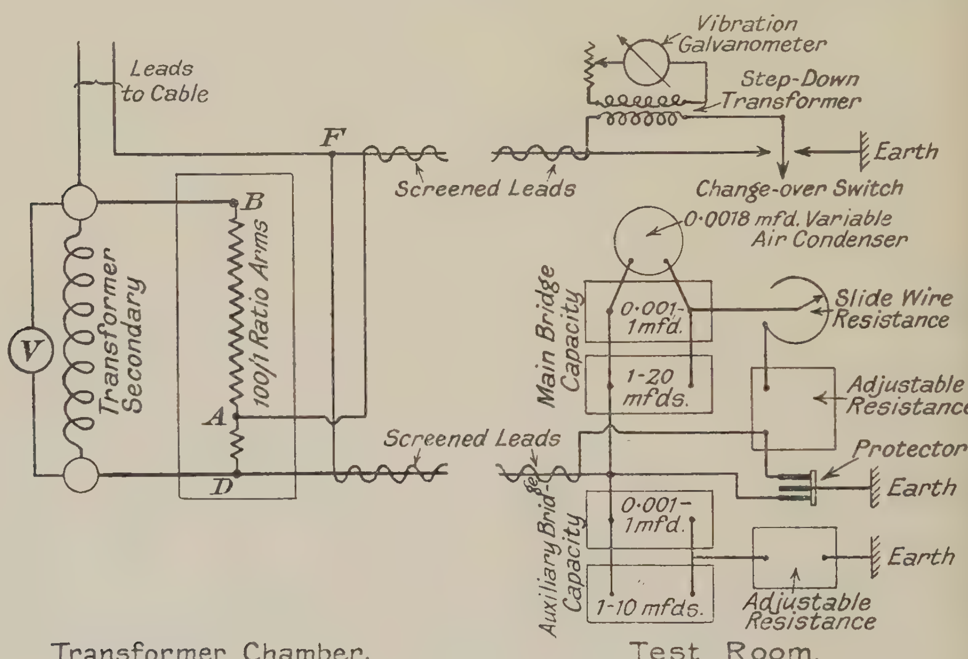


FIG. 5.—ARRANGEMENT FOR ROUTINE TESTING OF CABLES.

a voltmeter transformer 6,000 V./110 V. was used, because it happened to be available, and this gave an increase of about ten times in the deflection.) An adjustable resistance was put in the circuit to reduce the deflection to a suitable value when the bridge was out of balance. Theoretically best results are obtained when the galvanometer is exactly tuned to the supply frequency. It was found that this made the balance very sensitive to speed variation, and further, the response to adjustments of the bridge was sluggish. As there was generally no lack of sensitivity, it was found better to work with the galvanometer adjusted to about 3 cycles from the working value ; even so, for the higher voltages the deflections were very large, and it was usually advisable for rapid working to cut them down by means of the series resistance.

'rotective Devices.

Under normal working conditions no part of the apparatus to be adjusted is at higher potential than 300 volts (A.C.) from earth; if, however, the dielectric of he test condenser breaks down, then the full voltage appears on the measuring ondenser. This was guarded against by connecting the points F and D through park gaps to earth. A protector designed by the Post Office for guarding their erial lines against lightning discharges was used very successfully. It consisted f carbon blocks mounted on a porcelain base and separated by thin sheets of mica ierced with holes. This acted as a perfect insulator until the P.D. rose to about 50 volts, at which value the air broke down and a low resistance arc was formed; his was capable of passing enough current to operate an overload release in the rimary circuit of the transformer.

Determination of Corrections.

In Appendix I. the effect of the imperfections of the bridge arms is discussed, nd it is shown that it is only necessary to take into account the loss angle of the ariable condenser c, measured with the bridge ratio arms. This correction was etermined for the various mica condensers by comparison with a known air ondenser and a non-reactive series resistance placed in the arm BF, the rest of the ircuit being as usual; the capacity to earth of the leads served as the corresponding uxiliary bridge condenser. The P.D. of the transformer secondary was cut down o 200 V. (this being a safe figure for the air condenser) by using a 10 V. accumulator o excite the field of the alternator instead of the usual 110 V. mains. By employing xact tuning of the galvanometer, the power factor could be measured to within few per cent. On the large 1-20 mfd. condenser the values were of the order 0·001, nd on the 3 decade condenser about 0·0008. The correction is thus roughly 10% vhen measuring power factors of 0·01, and an error of as much as 10% in the deter-iination of the loss angle of the mica condensers would produce an error of only % in the power measurement.

It will be seen that the assumption is made that the constants of the bridge rms do not alter as the voltage is raised. This was checked by direct comparison vith a simple air condenser constructed as follows :—

The two plates were formed of thin sheets of tinned iron 6 ft. × 2 ft. 6 in. fixed n to flat wooden frames. The L.T. plate was cut into two portions by a con-inuous slot parallel to and about 4 in. from the sides, the sharp edges being turned lown. The upper (H.T.) plate was supported by porcelain insulators standing on he outer ring, which served as a guard plate and was earthed; thus there was nly air as dielectric between the plates of the condenser proper.

To assist in obtaining a balance, a coil having an inductance of 16 millihenries nd resistance of 20 ohms was placed in series with the 5,000 ohm ratio arm. With he plates about 2 in. apart, the capacity was 0·000197 mfd., and the bridge balance vas constant up to 12 KV., beyond which an increase in the series resistance r was ecessary. On altering the distance to $4\frac{1}{2}$ in. by using bigger insulators, the capacity vas reduced to 0·0000828 mfd., and the balance was undisturbed up to 25 KV. t is evident that no change was occurring in the bridge arms, the alteration being ue to ionisation of the air in the H.T. condenser. Similar results were obtained sing both air and mica condensers as the adjustable capacity c, and it was con-luded that the bridge constants do not alter with voltage.

From the results of these tests, the phase angle of the ratio arms (α) was calculated as $\alpha = \frac{\omega L}{R_2} - \omega cr$, L being the inductance (16 mh.) in series with R_2. When the capacity of the known air condenser c was 0·00828 mfd., the series resistance at 50 ∿ was 400 ± 10 ohms; thus $\alpha = -0{\cdot}00004$, which is less than the experimental error. This result shows that the unit method of constructing the ratio arms is very satisfactory.

Measurements on Cables.

In a single-core cable, the conductor forms one plate of a condenser and the outer lead sheath the other. The core is connected to the high potential point B on the bridge, and the sheath to the low potential point F. A second length of cable capable of withstanding the voltage, forms a suitable condenser C' for the auxiliary bridge, the sheath being put to earth. The sheath of the cable being tested must not be earthed, but the insulation need not be of a high order, e.g., satisfactory measurements were obtained on a coil wound on a dry wooden drum.

Multicore Cables.

Consider a cable containing n cores insulated from each other and from the

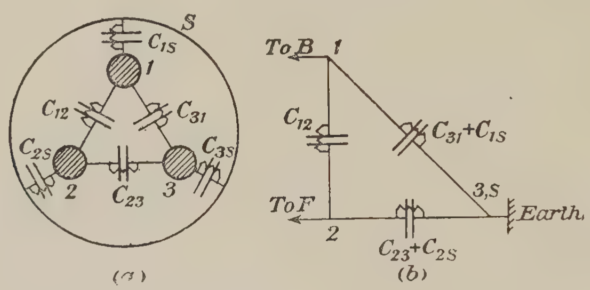


Fig. 6.—Capacities in 3-Core Cable.

metal sheath surrounding them. The "wire to wire" capacities between them may be represented by a series of $\frac{1}{2}n(n-1)$ leaky condensers connecting every pair of wires, and the "wire-to-sheath" capacities by a further n condensers connecting each core to the sheath.* A three-core cable is shown diagrammatically in Fig. 6(a). The Wagner double bridge enables us to determine the loss in, and capacity of any of these condensers directly.† To measure C_{12}, core 1 is connected to B in the bridge, and core 2 to F, the remaining core being joined to the sheath, which is earthed; the resulting system of condensers is shown in Fig. 6(b). The capacity of 1 to earth is $C_{31}+C_{1S}$, and this forms the condenser C' in the auxiliary bridge circuit. When final balance is obtained core 2 is brought to the same potential as S, and thus the capacity of 2 to earth, viz. $C_{23}+C_{2S}$ does not affect the main bridge; condenser C_{3S} is short-circuited, since 3 is joined directly to S. We have therefore eliminated all the capacities but the one to be measured, viz., C_{12}. Similarly to determine a

* A. Russell, Alternating Current Theory, Vol. 1, Chap. 4.
† K. W. Wagner, E.T.Z., Vol. 25, p. 635 (1912).

' wire-to-sheath " capacity, say C_{1S}, we connect 1 to the high potential point B, he sheath to F, and the other cores to earth. It is necessary for this test, as when aking a single core cable, that the sheath should be insulated from earth.

Cable Sheath Earthed.

If the cable sheath is already earthed, as when the coil is immersed in water, t is not possible to obtain any " wire-to-sheath " measurements in this way, and we ave to fall back on the ordinary single bridge. We can allow for the earth impedance of the bridge arms and of the leads to the cable, if we first take a reading vith the cable disconnected ; this gives the value of the resultant earth impedance magined all located in the arm BF. In what follows, the earth impedance of the oridge arms is understood to be included in the term " leads."

Let C_l, θ_l be the capacity and loss angle respectively of the leads.

C_c, θ_c be the capacity and loss angle respectively of the cable.

C_{c+l}, θ_{c+l} be the capacity and loss angle respectively of the leads and cable.

Then $C_c = C_{c+l} - C_l$,

nd $\tan \theta_c = (C_{c+l} \tan \theta_{c+l} - C_l \tan \theta_l)/(C_{c+l} - C_l)$.

With the existing double bridge arrangement we can compensate for the leads nd measure the cable directly as follows :—

With the cable disconnected and the arm FD (Fig. 7) open, the auxiliary bridge

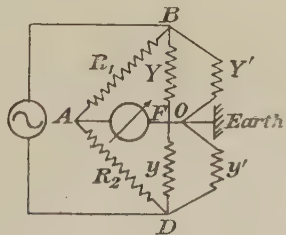


FIG. 7.—CABLE SHEATH EARTHED.

rm OD is adjusted to balance the leads. Then when the cable is on, the main bridge ondenser and resistance are adjusted for balance, and the readings give the capacity nd loss angle of the cable in the usual way.

For, let Y' represent the admittance of the leads, which is balanced by y'.

$$\therefore \frac{R_1}{R_2} \cdot Y' = y'.$$

Again Y and Y' in parallel are balanced by y and y' in parallel.

$$\therefore \frac{R_1}{R_2} (Y + Y') = y + y'.$$

Hence $\frac{R_1}{R_2} \cdot Y = y$, or y is the required balance for Y.

Loss in a 3-core Cable.

The dielectric loss in a 3-core cable is given by

$$W=3\omega(V_w^2 \tan \theta_w C_w+V_e^2 \tan \theta_e C_e)$$

the subscripts w and e referring to "wire-to-wire" and "wire-to-sheath" quantities respectively; when the voltages are balanced, $V_e=\frac{V_w}{\sqrt{3}}$, V_w being the line pressure.

Two tests were employed to determine W: (1) the three cores against sheath, at a voltage $\frac{V_w}{\sqrt{3}}$, giving a loss W_e; it the sheath were earthed, the single bridge method as described in the previous section, was used; (2) a "wire-to-wire" test with the double bridge, at the full voltage V_w, giving a loss W_w; then $W=W_e+3W_w$.

Routine Work.

From the apparently complicated series of adjustments required to obtain a balance using the double bridge, it might appear that this method is not so suitable for routine testing when time is a consideration, as the simpler wattmeter. This is not the case. It is shown in Appendix II. that, because of the high ratio of R_1 to R_2, a balance is quickly obtained, and only one cycle of adjustments, i.e., one of the auxiliary and then one of the main bridge, is needed for each subsequent step in voltage. Further, as was pointed out in the preliminary discussion of the Wien bridge, the balance is much less affected by fluctuations of speed and voltage than a wattmeter.

When testing cables, a reading was first taken at a low voltage (1 or 2 *KV.*). This indicated if the circuits were correct before the high tension was applied, and provided a datum for determining the change in the capacity and loss angle with voltage. As a rule, a series of readings at 4 or 5 different pressures was taken in addition to the working voltage, as not much extra time was occupied in obtaining the subsequent balances.

The following is an example of a "wire-to-wire" test on a 22,000 volt 3-core cable at 50 ∾. Ratio=100 : 1.

(1) Voltage V kilovolts.	(2) Capacity, c mfds.	(3) Resistance reading ohms.	(4) Resistance, r ohms.	(5) ωcr.	(6) $\tan \theta$	(7) Loss $=V^2\omega C \tan \theta$ watts.
1	0·7905	40·5	40·7	0·0101	0·0109	0·027
10	0·7905	40·5	40·7	0·0101	0·0109	2·70
14	0·7905	40·5	40·7	0·0101	0·0109	5·30
18	0·7907	42·1	42·3	0·0105	0·0113	9·10
22	0·7911	44·1	44·3	0·0110	0·0118	14·2
25	0·7915	46·0	46·2	0·0115	0·0123	19·1
30	0·7922	49·7	49·9	0·0124	0·0132	29·6

Column (4) is obtained from (3) by adding 0·2 ohm, the resistance of the connecting leads to the variable arm. Column (6) is obtained from (5) by adding 0·0008, the loss angle of the measuring condenser. These results are plotted in Fig. 2, where, however, the resistance figures are adjusted to include the effective series resistance of the condenser.

Comparison with Wattmeter.

Comparative tests were carried out using a Duddell-Mather dynamometer wattmeter with the scheme of connections shown in Fig. 8. The effect of the capacity current flowing between the series resistance and its metal containing tank was eliminated by joining the case to the junction of the wattmeter coils. Correcting only for the loss in the current coils, the results, when measuring power factors

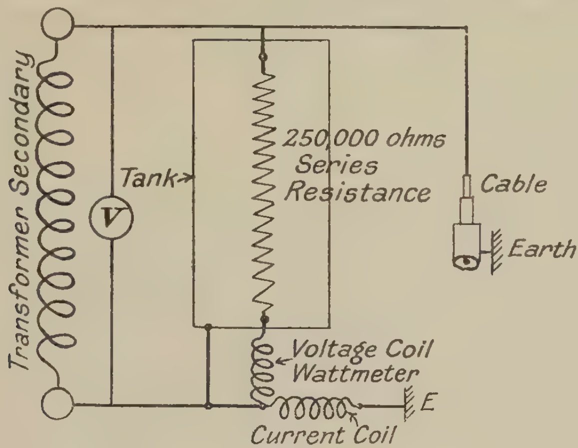


Fig. 8.—Wattmeter Connections.

of the order 0·01 agreed with the bridge figures to within 3 per cent., which was about the estimated experimental error due to unavoidable fluctuations in speed and voltage. An example is appended:—

Bridge. 50~. Ratio = 100 : 1.

Voltage *KV.*	Capacity, *c* mfds.	Total Resistance, *r* ohms.	ωcr	$\tan\theta$	Loss watts.
12·7	8·103	3·40	0·0086	0·0096	39·5
22·0	8·117	4·17	0·0106	0·0116	143

Wattmeter 50~.

Voltage *KV.*	Constant, watts per division.	Reading of torsion head divisions.	Reading less lead correction divisions.	Loss watts.	Loss in current coils watts.	Loss in cable watts.
12·7	12·5	3·65	3·27	40·9	2·5	38·4
22·0	12·5	13·8	12·3	154	7·5	146·5

General Remarks.

The bridge arrangement described in this Paper is the result of an attempt to devise an alternative to the wattmeter for measuring the dielectric losses in cables using only such apparatus as was already available, or could be easily made. It

has proved more successful than was originally anticipated, in particular regarding the accuracy and ease of balancing. Whilst it is most suitable for the purpose for which it was designed, it has shown a surprising degree of flexibility and has proved capable of measuring a large range of capacities of varying power factors, at low and high voltages, without alteration of the connections.

In the measurement of dielectric loss, the accuracy obtainable is limited by the closeness with which the frequency can be controlled and measured. For ordinary work, 1 per cent. was considered sufficient, and with special care, an accuracy of one-fifth per cent. has been obtained.

Arising from the use of the double bridge, the method is particularly adapted to the application of the "guard-wire" principle, e.g., the "wire-to-wire" tests of cables, the elimination of lead corrections, and the measurement of condensers with "guard-plates." This, combined with the ability to deal with small capacities, of the order 0·0001-0·001 mfd., has proved very useful in the testing of dielectric materials in small quantities.

It should be noticed that one pole of the condenser tested is brought to earth potential; this is a disadvantage when taking a "wire-to-wire" test on a cable, as the voltage is limited to the safe "wire-to-earth" pressure; on a three-phase system with earthed neutral, the W/E voltage is about $\frac{1}{\sqrt{3}}$ of the W/W voltage.

In conclusion, the author desires to express his thanks to the management of Messrs. Siemens Bros. & Co., Ltd., Woolwich, for permission to present this Paper; to Mr. B. R. Chaplin, for his help in the construction of apparatus, and in particular, to Mr. E. A. Beavis, A.C.G.I., B.Sc.(Eng.), A.M.I.E.E., for his co-operation both in that part and in the experimental portion of the work.

Appendix I.

Corrections for Imperfections of Bridge Arms.

The possible departures from the assumptions made in obtaining the equations for balance of the bridge are:—

(1) The time constants of the ratio arms may not be the same, in which case $\frac{R_1}{R_2}=m\angle\alpha$, where α is a small angle.

(2) The condenser used as the balancing capacity c may have a small loss angle β.

(3) The series resistance r may have a small phase angle γ.

Then
$$Z_4=r\angle\gamma-\frac{j}{\omega c\angle\beta}=r+\frac{\beta}{\omega c}-j\left(\frac{1}{\omega c}-\gamma r\right).$$

Since r is small compared to $\frac{1}{\omega c}$ and γ is small compared to $r, \gamma r$ may be neglected in comparison with $\frac{1}{\omega c}$, i.e., the phase angle of the series resistance does not introduce any appreciable error.

Further, $$Z_3 = m\angle\alpha \,.\, Z_4 = m(1+j\alpha)\left(r+\frac{\beta}{\omega c}-\frac{j}{\omega c}\right) = m\left(r+\frac{\alpha+\beta}{\omega c}\right) - j\,.\,\frac{m}{\omega c},$$

neglecting αr and $\frac{\alpha\beta}{\omega c}$ in comparison with $\frac{1}{\omega c}$.

Thus the corrections amount to adding a small quantity $\frac{\alpha+\beta}{\omega c}$ to the value of the series resistance r. In the expression for the capacity C, r itself enters only as a small correcting factor, so that the effect of the angles α and β can be here safely ignored. For the loss angle, we have

$$\tan\theta = \omega c\left(r+\frac{\alpha+\beta}{\omega c}\right) = \omega c r + \alpha + \beta.$$

The correction consists simply in adding the constant $(\alpha+\beta)$ to the loss angle as determined in the usual way; $\alpha+\beta$ may be regarded as the loss angle of the measuring condenser in terms of the particular ratio arms used.

Appendix II.

Theory of Double Bridge.

The general network of admittances resulting from the double bridge is shown in heavy lines in Fig. 9(a). To obtain the effect of the parallel system BOD with

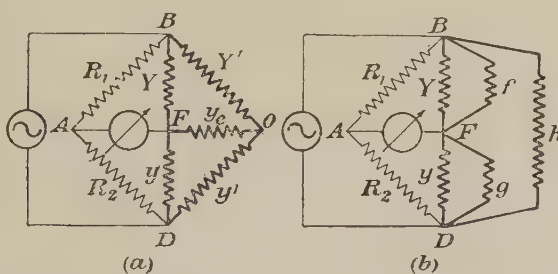


Fig. 9.—Transformation of Double to Single Bridge.

the "link" admittance OF when balancing BFD, we can transform the "star" OB, OF, OD, into its equivalent "mesh" as shown in Fig. 9(b).* The arrangement reduces to a simple single bridge; the admittance f is in parallel with the unknown arm Y, g is in parallel with the known arm y, while h is a shunt to the source and does not affect the balance.

$$f = \frac{y_e Y'}{Y'+y'+y_e} = pY' \text{ where } p = \frac{y_e}{Y'+y'+y_e}$$

$$g = \frac{y_e y'}{Y'+y'+y_e} = py'.$$

* S. Butterworth, Proc. Phys. Soc., Vol. 33, p. 315 (1921).

If the bridge ratio is m, the net effect is to add $g-mf$ to the known arm, i.e., $p(y'-mY')$. The conditions for no effect are (1) $y_e=0$, in which case the auxiliary circuit is not necessary, (2) $y'=mY'$, which is obtained when the bridge is finally balanced.

If all the admittances consist of capacities of fairly small phase angle, p becomes approximately $\dfrac{c_e}{C'+c'+c_e}$, where c_e is the "link" capacity; (when the bridge is balanced, $c'=mC'$ and $p=\dfrac{c_e}{(m+1)C'+c_e}\Big)$.

Now $y+py'=m(Y+pY')$.

As an approximation p is assumed unaffected by small changes in y'.

$$\therefore \qquad y+py'=\text{constant},$$

i.e.,
$$\omega^2c^2r+j\omega c+p(\omega^2c'^2r'+j\omega c')=\text{constant},$$

whence
$$c+pc'=\text{constant},$$
$$r+p\left(\frac{c'}{c}\right)^2 r'=\text{constant}.$$

Thus small changes $\delta c'$, $\delta r'$ in c', r' respectively, produce the same effect on the balance of the main bridge as variations $p\delta c'$, $p\left(\frac{c'}{c}\right)^2\delta r'$ in c, r respectively. Conversely, when balancing the auxiliary bridge, to obtain the effect of small changes in c and r, multiply by p', $p'\left(\frac{c}{c'}\right)^2$ respectively, where $p'=\dfrac{c_e}{C+c+c_e}$.

A given "out-of-balance" in the main bridge capacity will give rise to $\dfrac{1}{p}$ of the error when adjusting the auxiliary arm, and this again, when we return to the main bridge, will cause a new "out-of-balance" $\dfrac{1}{pp'}$ of the original value. Thus, in one cycle, we have reduced the error by the over-all factor pp'; in the same way, the over-all reduction factor per cycle for the resistance is also pp'. This shows that, to facilitate working, the "link" capacity c_e should be kept as small as possible, and also the advantage of using a large value for the ratio m.

In a particular 3-core cable, the "wire-to-wire" capacity was 0·007 mfd., and the "wire-to-earth" capacity 0·0275 mfd. per core. When taking a "wire-to-wire" test, the calculated figures allowing for the capacity of the leads, were as follows :—

$C=0{\cdot}00700$ mfd., $c=0{\cdot}700$ mfd., $C'=0{\cdot}0368$ mfd., $c'=3{\cdot}68$ mfd., $c_e=0{\cdot}0463$ mfd. Main bridge balance,

capacity factor,
$$p=\frac{0{\cdot}0463}{0{\cdot}037+3{\cdot}68+0{\cdot}046}=0{\cdot}0123\,(0{\cdot}016)$$

resistance factor,
$$p\left(\frac{c'}{c}\right)^2=0{\cdot}0123\left(\frac{3{\cdot}68}{0{\cdot}70}\right)^2=0{\cdot}34\ (0{\cdot}33).$$

Auxiliary bridge balance,

capacity factor, $$p'=\frac{0\cdot0463}{0\cdot037+3\cdot68+0\cdot046}=0\cdot062(0\cdot05)$$

resistance factor, $$p'\left(\frac{c}{c'}\right)^2=0\cdot062\left(\frac{0\cdot70}{3\cdot68}\right)^2=0\cdot0022(0\cdot0033).$$

Over-all reduction factor, $pp'=0\cdot00076(0\cdot00080)$.

The figures in brackets are the measured results, and these agree closely enough with the calculated figures to prove the approximate theory given above. In this case, the capacity was read to the nearest 0·0001 mfd., and the resistance to the nearest 0·1 ohm ; hence the permissible error in balancing the auxiliary circuit was $\frac{0\cdot0001}{0\cdot016}=0\cdot006$ mfd., and $\frac{0\cdot1}{0\cdot33}=0\cdot3$ ohm. The penultimate adjustments on the main bridge could be to within $\frac{0\cdot0001}{0\cdot0008}=0\cdot125$ mfd. and $\frac{0\cdot1}{0\cdot0008}=125$ ohms without causing error, i.e., 0·062 mfd. and 62 ohms from the final values. Actually, the maximum variation with the usual voltage steps never approached these figures, so that once the first balance was obtained, only one cycle (i.e., one adjustment of the auxiliary and then one on the main bridge) was needed for each step in voltage.

DISCUSSION.

Dr. E. H. RAYNER : This description of some of the methods available for what is an increasingly important branch of electrical measurements is very useful to have on record. I should like to point out one or two practical details more as a warning than a criticism. The author suggests a series of iron-cored inductances for a potential divider. It is to be remembered that the important region of phase angle in this kind of work is from about 2 deg. to zero, and any want of accuracy or similarity in subsidiary apparatus may become of first importance. I should not trust to any iron-cored inductances with joints in the iron. Our experience with nominally exactly similar potential transformers, which might make excellent potential dividers, is to the effect that their phase-angles may differ in the ratio of 3 to 2, and I am informed that the value can be made to vary by a hammer blow which may open or close the iron joints.

Another point to beware of is distributed capacity in the 500,500 ohm potential divider, either from the sections to earth or between sections. The same applies to the series resistance of the Duddell wattmeter. At 50 ∾ it may be negligible ; but it is one of the most difficult technical points to deal with, and the use of condensers, which are much more " pure " than resistances or inductances, is always a great point in their favour when phase angles of minutes or seconds are of importance.

Mr. L. HARTSHORN : Having made many measurements of dielectric losses with the Wien Bridge and the Schering Bridge, I was surprised to find that Mr. Rosen had used the Wien Bridge for high voltage tests, instead of the Schering one. Grebe and Zickner have shown that the Schering Bridge is capable of giving great precision under suitable conditions, and the saving of time in doing away with the necessity of an auxiliary balance is very considerable. It seems unlikely that it is more difficult to make a condenser to stand the high voltages than a resistance. Could not the condenser constructed out of sheet iron by the author be used for this purpose ? As the resistance arms can be kept very small and yet the sensitivity remains quite adequate at such high voltages, the effect of the earth impedances on the low-tension side should not prove to be serious, provided it is made as small as possible by a suitable arrangement of the apparatus. An important advantage of the Schering Bridge is that it requires very little power. The resistance arms carry but small currents, so that they can be small and shielded, whereas the large resistances used by the author will, as Dr. Rayner pointed out, have considerable distributed capacity to earth, and as the Wagner earthing device only compensates for capacities to earth concentrated at the corners of the bridge, this bridge is not likely to be entirely free from

error due to earth impedances. If comparative tests could be made, workers on dielectric losses would be glad to know how the two bridges compare as regards accuracy. In the matter of making the adjustments the Schering Bridge is unquestionably simpler.

AUTHOR'S reply (communicated): In reply in Dr. Rayner, I appreciate that it would be difficult to obtain two separate iron-cored inductances with the same phase-angle. However, what is suggested in the Paper is that the two ratio coils should have a common iron circuit; the effect of joints in the iron should then be the same for both. Further, the phase-angle of the ratio coils is easily measured and allowed for. This applies also to the resistances used as ratio arms. It is realised that the Wagner method of eliminating earth capacity is not perfect, but the resistances are connected in exactly the same way when their phase-angle ratio is measured, so that the correction takes account of any residual earth capacity.

In reply to Mr. Hartshorn, the objection to the Schering bridge is that the impedance between the L.T. pole of the condenser tested and earth must be of a high order. This is not always practicable in cable testing, whereas successful tests have been obtained with the arrangement described in which the resistance between the lead sheath and earth was only a few hundred ohms.

XXIV. *On a Coil-Galvanometer of Rapid Indication.* *By* DR. W. J. H. MOLL, *University of Utrecht.*

RECEIVED MARCH 6, 1923.

ABSTRACT.

The galvanometer is designed to secure rapid indication and steadiness of reading without unduly sacrificing the sensibility. The coil is long and narrow, and therefore of small moment of inertia: the mirror is supported by the wires forming the coil, between which it is slipped, and the coil is supported between an upper and a lower vertical wire, as distinct from strips, made of silicium bronze and put in tension.

IN estimating the efficiency of a galvanometer it is customary to mention only its sensitivity—e.g., expressed as the deflection given by 1 microvolt at 1 metre.

In practical work, however, the interesting point to the physicist is chiefly what is the least measurable P.D.—a quantity which enters into the precision of every reading? In the light of this criterion, the stability—i.e., the steadiness of the spot—is as important as the sensitivity.

A property of no less significance is the degree of rapidity of indication. As a matter of fact, the influence of this quantity on the precision of measurements is usually undervalued. In all galvanometer work disturbances of electrical origin are present, and their effect on the reading will in any case increase with the time during which they are active. Rapid indication, therefore, reduces the influence of such disturbances, and thus implies enhanced precision. The attendant economy of time is only a secondary advantage.

In the following will be pointed out the means by which it has been found possible to diverge from the usual construction of the coil-galvanometer, so as to obtain

(*a*) More rapid indication;

(*b*) Greater stability.

(*a*) The rapidity of indication depends on the moment of inertia of the coil, the directive force of the suspension, and the magnetic field (as it affects the damping). These quantities, however, also determine the sensitivity. We will inquire whether it is possible to increase the rapidity without reducing the sensitivity.

When we confine ourselves to the case of aperiodicity, and neglect air-damping, the following three relations apply—

$$P=\frac{HF}{DR} \qquad T^2=\pi^2\frac{Q}{D} \qquad 4DQ=\frac{(HF)^4}{R^2}$$

where P is the sensitivity, T the semi-period on open circuit, D the restoring couple, Q the moment of inertia of the coil, F the total area of its windings, H the intensity of the magnetic field, and R the resistance of the circuit.

Between these seven quantities there exist still other relations, varying with the construction of the instrument. The equations will, therefore, only serve to indicate the direction in which we have to seek.

We may simplify the problem as follows: Suppose, for simplicity, the coil to be circular, and the number of its windings as well as the material from which it is

made to be fixed. Assume, furthermore, a fixed relation between R and the resistance of the coil. In this supposition, R, F and Q are simple functions of the coil's diameter A, $R \propto A$, $F \propto A^2$, $Q \propto A^3$.

Or, dropping various constant factors and eliminating A

$$R=Q^{1/3}; \qquad F=Q^{2/3}.$$

By means of these relations we derive from the original expressions

$$P^4=\frac{Q^{1/3}}{D^3}; \quad \text{and} \quad T^2=\pi^2\frac{Q}{D}$$

These equations show very clearly that what we have to do is to decrease Q. This

FIG. 1.

will reduce T, but hardly affect P. By decreasing both Q and D, but D to a lesser degree, it will be even possible *to reduce T, while at the same time P increases.*

It may seem strange that this rather obvious procedure for reducing the moment of inertia of the coil, though repeatedly suggested, has never been applied in full consequence. Probably the reason for this is a difficulty which arises when the principle is applied. In the usual construction the coil is suspended by a strip, while the current is led off by means of a metal band so slack as to have no appreciable share in the directive couple. Now when the moment of inertia of the coil is made very small, and its weight consequently very slight, it will suffer from

inevitable vibrations to a much higher degree than would be the case with a heavy coil.

(*b*) The desired stability has been obtained in a rather obvious way—namely, by simply stretching the coil between two metal wires, instead of suspending it.

One might suppose that this tension would increase the directive couple ; but this is not the case. The directive couple is independent of the tension, and by using thin wires it may indeed be made as small as desired.

For this purpose wires are preferable to strips. As a matter of fact, strips, though generally used in galvanometers, are unsuitable. They are never absolutely straight ; the result is that a strip when stretched (in the usual construction, by the weight of the coil) will undergo asymmetric deformation. Any external impulse will cause a temporary change in the tension, and may cause a slight deformation of the strip, resulting in oscillation of the system. In point of fact, the instrument

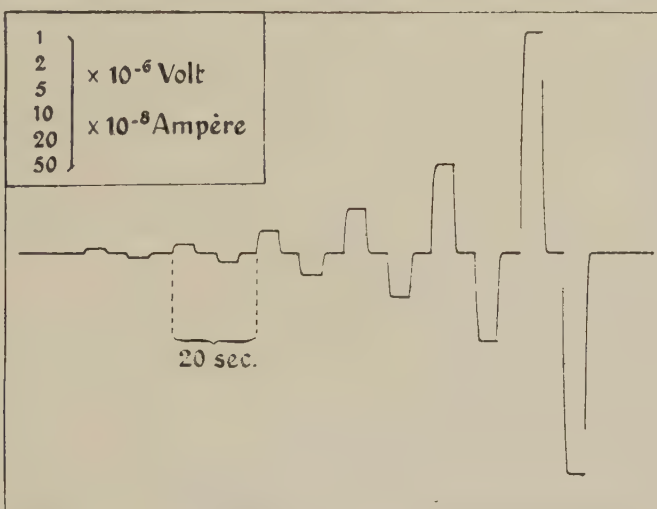


FIG. 2.

proved to be much more indifferent to vibrations after the strips had been replaced by wires.

These considerations have led me to the construction of a new type of galvanometer. The principle of the moving part is shown in the accompanying sketch (Fig. 1). The coil, the mirror and the stretched wires are marked *C*, *M* and *W* respectively. Great care has been devoted to the attainment of symmetrical distribution of mass with respect to the axis of rotation. It proved possible to make the instrument practically indifferent to the vibrations of a solid wall-bracket.

The coil is made of copper wires about 0·07 mm. thick, free from iron and enamelled ; it measures 5×40 mm., consists of 50 turns, and has a resistance of about 30 ohms. It has no former, but by means of a little shellac the windings have been made fairly rigid.

For the suspension-wires I use silicium-bronze, which has the same elastic

qualities as the usual phosphor-bronze, but a lower specific resistance. The wires were 0·015 mm. thick.

The arrestment of the stretched coil presented some difficulty. A special mechanism has been designed in which, by turning a button, the coil is caught at front and back simultaneously and clamped.

A single permanent magnet yields a field of about 700 gausses, strong enough to reach aperiodicity when the outer circuit has a resistance of 50 ohms. By means of a magnetic shunt this field may be weakened, so as to make the indications dead-beat for smaller values of the external resistance. I have also designed for this galvanometer an electro-magnet; for some purposes this is preferable to the permanent magnet type, as it allows the exciting current (and thus the damping) to be regulated from a distance, and also since a stronger field is obtainable. The current is intended to be supplied by two or at most three small accumulator cells.

The reliability and the rapidity of this galvanometer may be judged by an inspection of Fig. 2. This is the full size reproduction of a photographically registered series of deflections and zero-positions. In obtaining this record the galvanometer was placed on an ordinary bracket fixed to the wall of my laboratory. The resistance of the galvanometer system was 45 ohms; with an external resistance of 55 ohms the magnetic field had been adjusted so that the galvanometer was dead-beat. In this circuit of 100 ohms, electromotive forces of 1, 2, 5, 10, 20, 50 micro-volts were applied, the circuit being closed for 5 seconds, opened during the following 5 seconds, then reversed, and so on. The distance between the galvanometer and the registering apparatus was about 70 cm. The deflections reach their final value within 2 seconds.

It may be of interest to add that the ratio of the period of the coil when on open circuit in zero field to that in the usual field of 700 to 800 gausses is as 100 : 97. The slight difference in the periods is to be ascribed to traces of iron in the coil. Much care had to be expended in the construction in order to reach such a low residuum of magnetic matter.

For Discussion *see* page 260.

XXV. *A Thermopile for Measuring Radiation. By* DR. W. J. H. MOLL, *University of Utrecht.*

RECEIVED MARCH 6, 1923.

ABSTRACT.

The thermopile is designed to be quick-reading and free from zero-errors, as well as sensitive. The cold junctions are in contact with metal masses which keep down their temperature, and in order that the hot junctions may have small heat capacity the bi-metallic strips composing the thermopile are made of plates of constantan and manganine silver-soldered along an edge, rolled in a direction parallel to the edge into thin foil, and then cut into strips perpendicular to the edge.

THE problem of how most efficiently to construct a thermopile has been repeatedly considered in the literature of the last 25 years. Without exception Papers dealing with this subject emphasise the necessity of high sensitivity.

When comparing, however, in practice, various types of thermopiles, it soon becomes obvious that there are still other qualities which are of paramount influence on the effectiveness of the instrument. The most important of these is the degree of stability of the zero. A second quality is the quickness, namely, the time elapsing

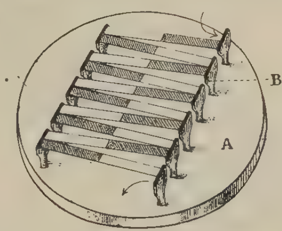


FIG. 1.

between the admission of radiation and the moment at which the full electromotive force is reached.

We shall describe a new method of construction which affords high stability and great quickness, together with high sensitivity.

In this method we have started from this principle: that it will be useful to minimise the heat-capacity of the active junctions, and to give a maximum value to that of the passive ones.

Fig. 1 is a diagrammatic sketch of the inner part of the new thermopile. *A* is a thick brass plate with two parallel rows of holes, in which flattened copper pins *B* have been clamped. A very thin coating of lacquer forms an insulating layer between these pins and the plate. On top of the pins the thermo-elements have been soldered. These consist of very thin blackened metal strips, made of the combination manganine-constantan.* With the exception of the first and last (indicated

* The combination manganine-constantan, though not showing a very high thermo-electric power (about 41 microvolts per 1°C.), has been chosen on account of the perfect elastic qualities of the two metals; these made it possible to give the thermo-elements their required thinness. An additional advantage lies in the durability of the metals, which neither rust nor corrode. For some applications it is, moreover, important that both manganine and constantan have a resistance which is independent of changes of temperature.

in the sketch by an arrow), all the pins bear two thermo-elements, and so form part of the passive junctions. The latter therefore possess a relatively rather high heat-capacity.

In order to minimise the capacity of the active junctions, the following method* has been introduced. Two rather thick plates of manganine (X) and constantan (Y) (*see* Fig. 2) are soldered together with silver. The superfluous silver projecting at both sides is polished away, after which a narrow seam of solder PQ remains. The bimetallic plate is then rolled out in the direction of the arrows. In this process the seam of solder is not broadened; with care it has been found possible to roll out this plate to a foil of only 0·005 mm. in thickness. By simply cutting this foil into narrow strips a large number of elements is obtained.

The plate A bearing the thermo-elements is tightly enclosed by a metal cover, provided with a transparent plate in front. All the necessary conditions have been thus fulfilled for obtaining both quickness and stability. The cover, together with the plate A and the pins B, form the "body," possessing a great capacity. In this body the thermo-elements and a little air are embedded. If no radiation is admitted, and no heat is conveyed to or carried from the body, the temperature

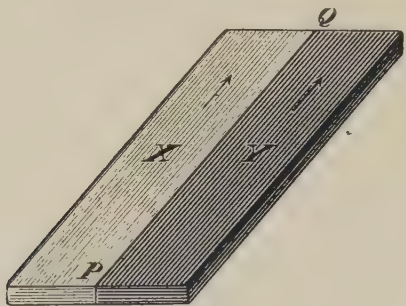


FIG. 2.

of the active and the passive junctions will be the same. As soon as radiation begins, the temperature of the strips and the surrounding air will rise above that of the body. Thermal equilibrium will result, the energy received being transferred to the body. Continuous radiation will tend to raise the temperature of the body, and increasing temperature of the room will have the same effect. It is a pronounced advantage of the construction that such temperature variations do not affect the thermo-electromotive force.

We may lay stress upon the fact that this result has been obtained (1) by giving to the body a large heat-capacity; (2) by keeping that of the strips and the surrounding air small; and (3) by securing an effective heat transport between the strips and the body. It is true that the means by which this transport is favoured (short strips, and small distance between the strips and the plate) are at the expense of the sensitivity. But we are convinced that quickness and stability are features so important that some sensitivity may be sacrificed. From the data which follow below it will be clear that the remaining sensitivity is still very high.

* This method is some years old; we gave a demonstration of these thermo-elements in 1913, during a Congress at Delft.

The new construction has the great advantage that it is not necessary to screen off the passive junctions; indeed the elements can be exposed to radiation along their full length. It is therefore possible to arrange the elements areally, and to construct surface-thermopiles.

Fig. 3 shows the grouping of 80 elements within a circle of 2 cm. diameter.

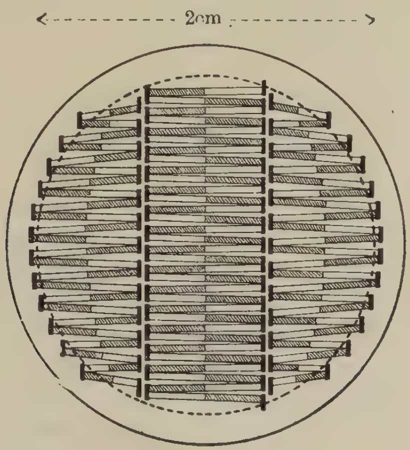


FIG. 3.

Such a thermopile has a resistance of about 50 ohms, and yields about 75 microvolts for the radiation of a candle at 1 metre, when closed by a plate of rocksalt.

The elements may also be arranged on a smaller surface; this increases the sensitivity per unit of area. Fig. 4 shows a type of construction, which in our investigations has largely fulfilled the requirements. This thermopile is closed by a window of fluorite having a circular opening of 6 mm., behind which 18 elements are arranged. Its resistance is about 25 ohms, and a candle at 1 metre generates

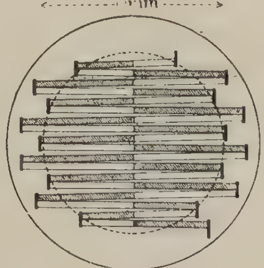


FIG. 4.

about 13 microvolts. By providing the thermopile with a conical reflector the sensitivity may be largely increased.

Both thermopiles are equally quick in response. If they are connected to a sufficiently rapid galvanometer (adjusted so as to have its indications dead beat), and radiation is periodically admitted and intercepted, the spot comes to a perfect standstill *within* 2 *seconds.*

It goes without saying that thermopiles built on this principle may be provided with a slit. In this case too the use of a fluorite window is recommended, in order to avoid air-currents.

A few examples of the working of this thermopile have been given in an earlier Paper, dealing with a registering microphotometer.* In this instrument the image of a strongly illuminated photographic plate is projected on to the slit of a small thermopile. The thermo-current is led to a rapid galvanometer, the deflections of which are photographically recorded. In that article some records of spectra were reproduced. From these the quickness and the accurate working of the thermopile will be evident.

DISCUSSION.

Mr. F. E. Smith commented on the strikingly dead-beat character of the indications given by both instruments. He suggested that the thermopile strips might be reduced to even a less thickness than ·005 mm. by solution in acid. The galvanometer seemed to have been designed to produce what, with a single fibre suspension, would have been an extremely sensitive instrument, but then the supporting wires were put in tension to reduce the period. Would it still be possible in these circumstances to obtain the required sensibility ?

Dr. R. T. Beatty recalled the time when it was necessary to work with a bismuth-iron couple of great heat-capacity and an insensitive Thomson galvanometer. Later came the Paschen astatic galvanometer ; but this instrument required such heavy protection from stray magnetic fields that few cared to use it. Dr. Moll's improvements should make possible an advance in measurements of the distribution of energy in the spectrum, for instance in the H and He spectra.

Mr. C. R. Darling suggested that the apparatus might usefully be applied to measure the distribution of energy in the spectrum of a glow lamp.

Mr. J. Guild inquired whether the present galvanometer differs materially from that exhibited by Dr. Moll to the Society three years ago, and what is the number of junctions per cm. in the linear form of the thermopile. If silicium bronze is similar in its properties to phosphor bronze the tension of the supporting wires of the galvanometer would have a serious effect on the sensibility of the instrument.

Mr. L. F. Richardson said that he thought Dr. Moll's thermopile admirable for indoor use. For work out of doors, where a rock salt cover could not be used and gusts of wind could not be entirely excluded, Mr. W. H. Dines had found it desirable to make the hot and cold junctions of as nearly the same thermal capacity as possible, so that air currents might affect both equally.

Mr. R. S. Whipple, in reply to the discussion, said that the difficulty of rolling the strips is immense, and Mr. Smith's suggestion as to solution in acid is well worth trying, though there might be difficulties due to unequal chemical action. He agreed that, although there is no instrument as sensitive as the Paschen galvanometer, there is none so troublesome to use.

* Proc. Phys. Soc., Vol. 33, Part 4, p. 207 (1921).

DEMONSTRATION of (a) A New Balance for Compensating the Temperature Error of Watches and Chronometers, and (b) A Centre-seconds Marine Chronometer with Electric Contacts. By PAUL DITISHEIM, *La Chaux de Fonds, Switzerland.*

(a) NEW BALANCE FOR COMPENSATING THE TEMPERATURE ERROR OF WATCHES AND CHRONOMETERS.

WHEN submitted to a rise in temperature, any uncompensated watch or chronometer fitted with the ordinary steel spring loses approximately 11 seconds per 1°C. in 24 hours. John Harrison first proposed a correcting curb method, so arranged that heat caused unequal expansion in two strips of brass and steel fastened together, the bending altering automatically the position of the pins and the effective length of the hair spring. At an early date, it was recognised that this correction should be effected by the balance-wheel itself, and not by the spring. In 1765 Pierre Le Roy accordingly applied the principle of the bimetallic rim, still used in chronometers and watches. This brass and steel compensating balance-whee was successfully improved by Arnold and Earnshaw. In this balance the action of increase of temperature on the elasticity of the hair spring is compensated by a corresponding reduction in the moment of inertia of the two circular bimetallic rims which curve in towards the centre of the balance-wheel.

In his Guthrie lecture* before the Physical Society of London, Dr. Guillaume presented a full account of the anomaly of the nickel-steels. By simply substituting a balance-wheel with nickel-steel and brass bimetallic rim, the Director of the International Bureau of Weights and Measures has solved the problem of the middle temperature error (Dent's error), the correction of which had been the subject of considerable endeavour and effort, especially among English watchmakers. The rates of high-class watches and chronometers were thereby greatly improved, the variation of rates now being only a quarter of the figures of 20 years ago. It should be noted that this Guillaume compensation balance can only be used with an ordinary steel hair spring.

The ordinary watch also benefits by use of the nickel-steel alloys. It can now be compensated solely through the action of a nickel-steel alloy substituted for the usual steel hair spring; there is no bimetallic rim whatever—we come back to the plain, solid, uncut balance ring. By this means the temperature-coefficient is reduced to less than one second per day per 1°C. About 50 million watches have been manufactured with such auto-compensating hair springs since 1897, when Dr. Guillaume and the late Paul Perret made the first trial experiments.

The so-called *Invar* spring could not, however, be applied to the highest class of watches; it had the disadvantage of causing a large middle temperature error, corresponding to a gain of 20 to 25 seconds per day, as compared with the rate at the extreme temperatures over an interval of 30°C. We should recall that an uncompensated watch would vary by five to six minutes per day in the same period.

A great advance, now to be recorded, consists in the use of hair springs made from Dr. Guillaume's new alloy *Elinvar*. The combination of this spring with the simple monometallic balance-wheel is now good enough to fulfil the requirements of higher-grade timekeepers.

* Proc. Phys. Soc., Vol. 32 Pt. 5 (1920).

Another advantage of the elinvar hair spring is that, when made from a casting appropriate to a determinate linear expansion of the balance ring, very uniform rates over a wide range of temperature are obtained, without need of adjustment by highly skilled timing watchmakers.

The first temperature test made in England with an elinvar spring was carried out at the National Physical Laboratory, Teddington, from March 16 to April 29, 1920, with Watch No. 44811 Paul Ditisheim. Since that time other observatory and workshop tests have also proved the efficiency of the system.

It is no exaggeration to state that the new gift made to the horological industry will bring about a revolution in watch manufacturing methods. Dr. Guillaume, however, did not see how this combination could be adjusted, and feared it could not be used where the very highest accuracy was required. I have here specimens of balances which I have designed to overcome the difficulty. The

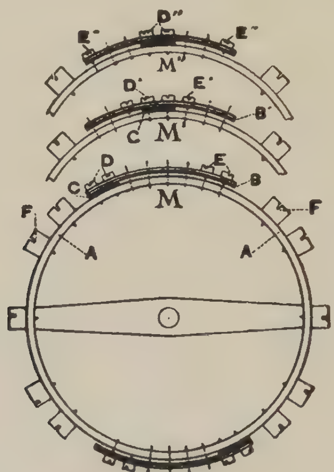


FIG. 1.—PAUL DITISHEIM COMPENSATING BALANCE, WITH MONOMETALLIC RING, FOR POCKET WATCHES.

A. Uncut monometallic ring. *B*. Compensating biblade. *C*. Removable heel holding the biblade. *D*. Fixing screws for the biblade. *E*. Compensating screws. *F*. Poising screws, without compensating influence—*M*, *M′*, *M″* varied arrangements of the screws and heel, showing the many facilities for adjusting the compensation through changing the position of the heel and the screws to *E*, *E′*, *E″*. (The aim has not been, as in Fig. 3, to reduce the barometric coefficient; the appearance is nearly the same as in the usual compensating balance.)

principle consists in fitting on two opposite points of its rim two small bimetallic plates, which enable the timers to touch up and adjust in the usual manner.

The tests I made of this combination have proved it to be very efficient, as was confirmed by the first-class pocket-chronometer trials to which it was submitted at Neuchâtel Observatory.

Some details of this new balance will now be given. The great practical interest of using the elinvar hair spring being precisely the suppression of the bimetallic balance, one may wonder wherein lies the advantage of the new combination, in which the bimetallic plate is reinstated. The difficulties normally encountered by

timers in setting up a balance are to combine at all temperatures both its compensating action and its poise. The first depends on the combined actions of the two bimetallic rims; the second on the continuous regularity of their action. Furthermore, in calculating the end curves of the hair spring it is necessary to take into consideration the action of centrifugal force on the shapes of the blades, the effect of which in the association of a steel hair spring with an ordinary compensating balance may account for a difference of as much as 12 seconds in a day, as between large and small oscillations; this action is reduced to about a fifth by Guillaume's balance.

Now the coefficient of linear expansion of the metal of the balance fitted with our compensating bimetallic blades can always be chosen so that the uncompensated residue is reduced to a quantity not exceeding one-hundredth of that present in the compensating balance used with a steel hair spring; and the perturbing influences which the timer has to overcome will be reduced in a similar proportion. The

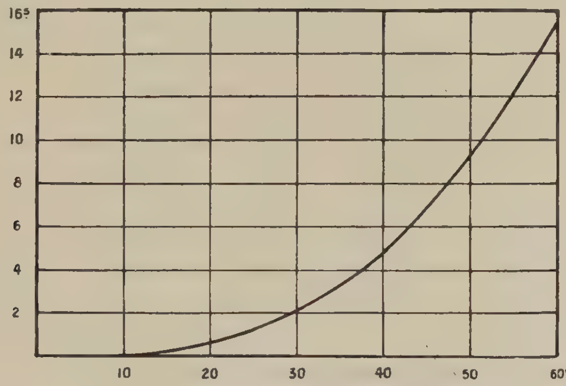


FIG. 2.—BALANCE WITH COMPENSATING AFFIX.

Abscissa: Angle subtended at centre of balance by biblades. Ordinate: Correcting effect in seconds per day.

poise at all temperatures is almost perfectly maintained by the monometallic rim of the balance, and in practice this result can easily be attained; the effect of centrifugal force on this part is almost negligible.

With regard to the balance itself (Fig. 1), the first part manufactured is a plain, monometallic ring, to which small blades made of two metals of different expansibilities are adapted, the most expansible being inside or outside according to the necessity to correct fast or slow time when at the higher temperatures. In selecting these metals, the value of the quadratic coefficient of their equation of expansion can be utilised in order to neutralise any secondary error which might arise from the use of the elinvar alloy. Other means available for regulating the action of the biblades are their length and thickness.

In order to avoid any protuberance, the two blades can be inserted into grooves made in the rim of the balance. (Fig. 3.)

The screws needed to attain the equilibrium (poise) of the balance and to regulate its moment of inertia are level with the surface of the rim. Some of them, being screened, may be given a dissymmetric shape, permitting their rotation for the last

touching up. This design has been adopted in order to reduce as much as possible air friction, which constitutes one of the main causes of variation in the timekeeping of watches now that the compensation has been so greatly improved, thanks to the Guillaume balance. In balances fitted with projecting screws the barometric coefficient is about 0·010 second per day for 1 mm. of mercury in the case of a chronometer of the "Marine" size, and about double this figure for one of the size of a large pocket watch. It can be gathered from observatory tests that timekeepers which have shown daily rates slightly inferior to one-tenth of a second would have lost their rank of classification had they been tested in periods of unstable atmospheric pressure. The experiments already made lead us to assign to the new balance a barometric coefficient about two-thirds of that of an ordinary compensating balance. This is a first attempt to reduce the barometric coefficient,

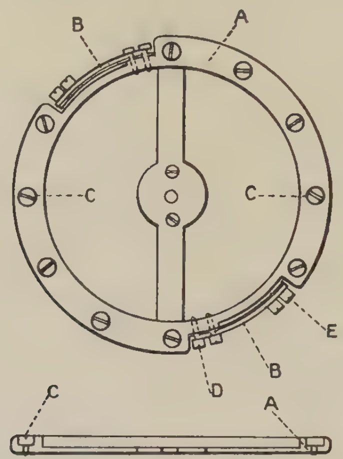


FIG. 3.—PAUL DITISHEIM MONOMETALLIC BALANCE, WITH COMPENSATING AFFIX.

A. Monometallic independent uncut ring. *B*. Bimetallic compensating segment. *C*. Poising screws inserted in the level of the rings. *D*. Screws for fixing the biblade. *E*. Screws for adjusting the compensation.

the consequences of which will become still more important as new improvements are effected in chronometry.

The total absence of the centrifugal force effect reduces the timing to the pure Phillips curves, and thus facilitates the attainment of isochronism. The use of wireless signals for the daily checking of chronometers makes it no longer an absolute necessity to have regular timing over a long period, but, on the other hand, increases the importance of perfect timing during a diurnal period. In other words, the question of isochronism has become now the most important problem for the maker of chronometers, and it seems that the combination of the elinvar hair spring and balance with monometallic rim will equally constitute elements of progress in this direction.

Another advantage presented by the new form of compensation lies in the practically non-magnetic and rustless qualities of the elinvar alloys. The many facilities offered by the use of small detachable biblades, whose components can

conveniently be changed or inverted, include the selection of a suitable non-magnetic metal or alloy for the larger uncut part of the plain balance-wheel above described.

(*b*) Marine Chronometer.

Two models of marine chronometer with large centre-seconds hand and electric contacts taken to a sounder were shown: (*a*) with lever escapement, five beats per second, only one of which is heard in the sounder; (*b*) with chronometer detent escapement. Once every five beats of the fast train lever escapement an electric contact is made, closing the circuit of a sounder by means of which a distinct tick is heard every second. At the sixtieth second one beat is omitted.

In an observatory the sounder can be fitted close to the transit instrument, the chronometer being at a point free from changes of temperature.

The mean variation of daily rate does not exceed 0·1 second.

XXVI. *A General Solution of the Problem of Finding the True Vertical for all Types of Marine and Aerial Craft.* By J. G. GRAY, *D.Sc., F.R.S.E., Cargill Professor of Physics, University of Glasgow.*

(SUMMARY OF A LECTURE DELIVERED JUNE 8, 1923.)

IN the first part of the lecture it was shown that the difficulties presented by this problem arise from the horizontal accelerations which result from the turning of vehicles. A gyroscopic pendulum to succeed must possess a real precessional period, or a virtual precessional period during turning motion of the vehicle on which it is mounted, which is measured in hours.

Pioneer forms of Gray stabiliser were described. These were devised by the author and his brother for use in the Royal Naval Air Service. A system composed of a single gyroscope (mounted with its axis normally vertical) and an erector connected rigidly to one another, is pivoted to a gimbal frame by means of two cross pivots ; and this frame is in turn attached by means of fore and aft pivots to uprights, or the equivalent, carried by the aeroplane. The system composed of the gyroscope and erector is mounted and balanced up so that the centre of gravity of the entire system coincides with the intersection of the pivot axes.

Various forms of erector made use of with these pioneer instruments were described, and their dynamical action discussed. One form consists of a circular track carried by the pivoted system, and so arranged that when the pivoted system is upright the track is horizontal. One, two, or more balls rotate on the track, each controlled by a pusher and a check carried by a member which rotates slowly (about 12 revolutions per minute) in the direction of spin of the gyroscope. When the system is upright the balls move round the track in contact with their pushers, and form a balanced system. When the system is inclined to the vertical the track is inclined to the horizontal, and each ball when ascending the slope of the track rests against its pusher, but after crossing the crest of the slope it is accelerated down the track and rests against its check. The motion of the balls relative to the pushers and checks results in the application to the pivoted system of an integral erecting couple.

It was shown how the contrivance could be arranged so that the stabiliser possessed the property that it was blind to the apparent vertical during turning motion of an aeroplane on which it was mounted, but conscious (so to speak) of the true vertical during the ordinary flight of the aeroplane.

These pioneer forms of the instrument were found to possess an accuracy, for bombing purposes, amounting to one-eighth or one-tenth of a degree, or about 20 feet on the ground from a height of 12,000 feet.

Previous work on the problem failed because the devices produced possessed the property that they left the true vertical quickly in the presence of the accelerations which accompany turning motions of the aeroplane, and returned to the true vertical only very slowly after the resumption of ordinary flight. The pioneer forms of Gray stabiliser (when properly used) moved only very slowly, if at all, towards the apparent vertical when the aeroplane was turning, and after the resumption of ordinary flight moved towards the true vertical (supposing an error to exist) relatively quickly.

Finally the latest forms of Gray stabiliser were described. These set themselves

into the true vertical even when the vehicles on which they are mounted are turning, and this holds for all speeds of turning. Control is never lost. This result is obtained by constructing the apparatus so that a horizontal component of spin lies across the pivoted system, that is, parallel to the cross pivots. The pivoted system is mounted so as to be pendulous with respect to the pivots, and the direction of the horizontal spin, and its amount, is arranged so that when the vehicle turns there comes into existence a gyroscopic couple, applied about the fore and aft pivots, which is exactly equal and opposite to the so-called centrifugal couple applied to the pivoted system. Both couples are proportional to the angular speed at which the vehicle turns, and both change sign with that of the turning motion. Hence the compensation is correct for all speeds of turning.

This type of stabiliser when fitted with special erectors, a simple example of which was described, supplies a complete solution of the vertical problem. Assuming an error made (small) in adjusting the compensating component of spin the resting position of the device is one in which it is inclined to the true vertical at a very small angle. The devices solve the problem completely for the rapid and slow turning movements of aeroplanes, and for the turning motions of ships.

DISCUSSION.

Sir JAMES HENDERSON, after complimenting Professor Gray on his lecture, referred to the great difficulties of the problem. These difficulties vary with the accuracy aimed at. If it be only required to maintain the vertical within one degree the problem is very easy, but if it be required to maintain it within one minute of arc the difficulties are very great, almost if not quite unsurmountable. The difficulties increase almost as the square of the accuracy aimed at.

Professor Gray had concentrated upon the problem of the bomb-dropping sight for aircraft, and had achieved considerable success during the war. The practical accuracy then aimed at in this connection was to maintain the vertical to about $\frac{1}{4}$ degree, and to this degree of accuracy Professor Gray had attained, as stated in his lecture.

The accuracy of any instrument of this type could be calculated accurately if the method of control were continuous. In Professor Gray's instrument the method of control involves a discontinuity which does not lend itself to easy computation. There are other methods of control, however, which are continuous and which enable accurate calculations to be made.

Professor Gray had said nothing in his lecture to give any idea of the amount of damping he employed. This determines the accuracy, because if the damping be very great the gyro follows the *virtual* vertical at corresponding rates and the deviation increases accordingly. On the other hand, if the damping be greatly diminished a considerable deviation arises from the rotation of the earth and a connection to the gyro-compass is required to compensate it. Thus when great accuracy is required the problem becomes excessively difficult.

Professor Gray's method of eliminating the effects of the centrifugal force during a turn was ingenious. If the speed adjustment be perfect, then any deviation which the pendulum has relatively to the vertical at the beginning of the turn turns in azimuth with the plane. The deviations due to any change of velocity either fore or aft or during a turn are, however, not so easily compensated.

M. PAUL SCHILOWSKY said that naval experts ask for a degree of accuracy which must be the despair of gyroscopic inventors, but accuracy to a degree or half a degree of angle is by no means unattainable. He had tried a different solution from that given by the lecturer, viz., that of suspending the gyroscopic pendulum in neutral equilibrium on knife edges passing through its centre of gravity. In time, however, precession takes place with this arrangement, and the pendulum errs progressively from the true vertical. What is required is that the pendular suspension should be neutral for curved motion but stable for rectilinear motion. Sir James Henderson has designed an instrument for indicating the curvature of the path of the aeroplane. A combination of the two devices should make it possible to meet the requirement mentioned. The lecturer's plan was open to the objection that very rapid changes of rotary speed in the gyroscope would be necessary to compensate for the rapidity with which an aeroplane changes

its linear speed. His admiration for the lecturer's work was sincere, and he should expect great results if the latter would take up the problem of altering the suspension of the gyroscope according to the curvature of the path.

Mr. T. SMITH said that all present would have followed the lecture with the greatest interest. It was not easy, however, for those less familiar than the lecturer with the practical handling of gyroscopic apparatus to appreciate all the points at issue in the absence of a formal mathematical treatment. Possibly mathematical difficulties would arise from the discontinuities which had been mentioned by Sir James Henderson, but the modern methods for dealing with discontinuous quantities developed in connection with the quantum theory might perhaps give a hint as to the line of attack to be adopted in the present case.

The LECTURER, in reply to the discussion, dwelt mainly on the point raised by M. Schilowsky, and claimed that his apparatus did become automatically "blind" during curved motion of the aeroplane in a manner satisfying M. Schilowsky's requirement. The effect of the earth's rotation on the gyroscope is very small provided the real precessional period is not great. The virtual precessional period, during turning motion, must of course be great.

The PRESIDENT congratulated the Lecturer on his solution of an important and difficult problem, and proposed a vote of thanks which was carried with acclamation.

XXVII. *The Fine Structure of Some Sodium Salts of the Fatty Acids in Soap Curds.*

By S. H. Piper, *D.S.O., B.Sc., F.Inst.P., and* E. N. Grindley, *B.Sc.*

Received June 8, 1923.

ABSTRACT.

X-ray photographs of certain sodium salts of the fatty acids (soap curds) show lines due to reflections from planes with very wide spacings of the order 40 A.U. These planar spacings increase uniformly with the number of CH_2 groups in the molecule, indicating an effective length of 1·25 A.U. for the CH_2 group. These and other lines can be accounted for by assuming that the curds are in the smectic state described by Friedel.

These experiments were originally undertaken to determine by X-ray methods whether the fibres observable in certain soap curds* were crystalline in nature. The observations are not yet complete, but some measurements already obtained appear to be of sufficient interest to merit publication.

Photographs were taken by the powder method, with a hot filament X-ray tube and camera modelled on those of Shearer. The wave-lengths used were those of copper K_α and K_β unfiltered. The watery nature of many of the specimens examined necessitated exposures of 2 to 4 hours with a tube current of 10 to 20 milli-amps. A thick layer of the substance has been used, usually about 2 mm., and in consequence the lines obtained are not usually sharp and fine, but measurements on the same plate of the best lines usually agree to about 1 per cent. or 2 per cent.

All the specimens examined show one very wide spacing of the order of 40 A.U. giving well defined lines. Up to five orders of these lines have been observed and measured. There are also two spacings of the order of 4 A.U. giving very black but diffuse lines. The wide spacing is in accord with the 43·5 A.U. observed by de Broglie† for some Oleates.

The principal salts and spacings observed are shown in Table I. The spacings are calculated from the best lines only.

Table I.

Salt.	No. of CH_2 groups.	Wide spacing.	Difference.	Close spacings.	
Na Laurate............	10	33·5 A.U.		4·22	4·88
			5·0 A.U.		
Na Myristate.........	12	38·5 A.U.		4·18	4·9
			5·0 A.U.		
Na Palmitate	14	43·5 A.U.		4·15	4·9

It will be observed that there is a regular increase in the wide spacing as the number of CH_2 groups in the molecule increases, and that the addition of two CH_2 groups to the molecule gives a constant difference of 5 A.U.

* Darke, McBain and Salmon, p. 395, P.R.S. 98 (1921).

† C.R., No. 11, Vol. 176, p. 738, March (1923).

According to Langmuir* the length of the Palmitic Acid molecule is 24 A.U. This is of the order of one-half the wide spacing shown by the Sodium Palmitate curd, and suggests that the molecules are distributed in planes to which their axes are perpendicular, and that the molecules in successive planes face in opposite directions. In this case the reflections are probably from parallel planes containing Sodium atoms and spaced at distances equal to twice the length of the molecule. The common difference of 5 A.U. would then be equal to the effective length of four CH_2 groups.

The diffuse nature of the lines given by the closer spacings does not allow of an accuracy of measurement greater than about 4 per cent. Langmuir (*loc. cit.*) gives measurements of the effective cross-section of the heads of the molecules of two fatty acids in thin films as 21×10^{-16}cm² and 22×10^{-16}cm². This is assumed to be the area of cross-section of the —COOH group. Our end group only differs in nature from this by the substitution of sodium for hydrogen, and as one of the oxygen

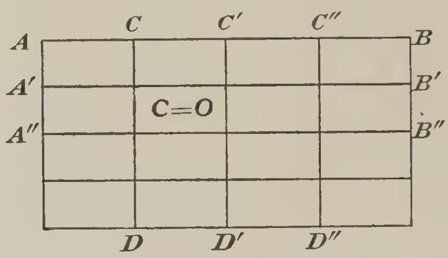


Fig. 1.

atoms is placed obliquely it is probable that —COOH and —COONa have approximately the same cross-section.

The distribution of the —COOH group is usually supposed to be $\underset{\underset{\displaystyle O}{\|}}{-C\text{-}O\text{-}H}$ and it is quite conceivable that such a group would have an effective width in its plane greater than in the direction at right angles to it. The products of the close spacings given in Table I. is approximately 21×10^{-16}cm², suggesting that the lines from which these spacings are calculated are due to planes containing the lengths of the molecules and perpendicular to one another. The small rectangles are the effective cross-sections occupied by the COONa groups in the widely spaced planes. The planes referred to above are shown in section as *AB* for the 4·2 spacing and as *CD* for the 4·9 spacing. The molecules lie perpendicular to the plane of the diagram. This is perhaps made clearer by Fig. 1. This value of 21×10^{-16}cm² agrees with that given by N. K. Adams† for condensed films of fatty acids.

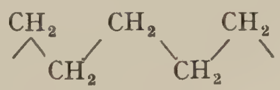
There are many other faint lines shown on the photographs which it has not yet been possible to analyse.

In addition to the three salts shown in the Table we have measured the lines given by sodium stearate both as a curd and in the dry condition. In this salt there are 16 CH_2 groups, and we should expect a wide spacing of about 48·5 A.U.

* J. Am. Chem. Soc., p. 1865, July-Dec. (1917).
† N. K. Adams, P.R.S., A., Vol. 101, p. 452 (1922).

Measurements of four plates give a mean value of 44 A.U., which is nearly that observed for the Palmitate. Langmuir (*loc. cit.*) gives 25, and Adams (*loc. cit.*) 26·2 A.U. as the length of the Stearic Acid molecule. Langmuir also gives 24 A.U. as the length of the Palmitic Acid molecule. There seems little doubt that the Stearic molecule is longer than the Palmitic, and we have not yet succeeded in accounting for our apparently anomalous values. We find that the values of the spacing as measured for the stearate show more variation among themselves than do the measurements on the strong lines of the other salts. There is also in the Stearate an indication of alteration in spacing as the amount of water in the substance changes. This substance is now being more thoroughly examined.

Assuming that these substances are in the smectic condition described by Friedel* (as is done by de Broglie† for the Oleates he measured), we can account for some of the properties of the smectic bodies. In many cases for bodies in the smectic state the molecules have a common direction and are distributed over parallel equidistant planes (planes of Grandjean) and have their lengths normal to these planes. The distance between the planes is assumed by Friedel to be equal to, or a multiple of, the thickness of the stratified layers of soap films measured by Wells‡, that is, about 43 A.U. On our theory this minimum thickness is the length of two molecules perpendicular to the plane and so placed that the CH_3 groups are in contact inwards and the sodium atoms on the two free surfaces. The affinity of the $—CH_3$ groups for one another is greater than the affinity of the sodium atoms; the planes containing the sodium are therefore planes of slip. The structure attributed to the $—CH_2—$ chain by Langmuir (*loc. cit.*) is of the type



and if several of these are placed parallel the chains can lock and give a rigidity to the structure in the direction of the length. This suggests fluidity in the direction of the planes and rigidity at right angles, as mentioned by Friedel (*loc. cit.* p. 302).

That this is the true nature of the structure of the chain also receives support from the following considerations.

Assuming the angle between the lines joining the centres of successive Carbon atoms in the zig-zag chain is 109°28′ as in the diamond, and taking 1·54 A.U. as the size of the carbon atom, the distance between the centres of successive carbon atoms measured along the axis of the molecule will be 1·26 A.U. Four carbon atoms will account for a length of 5·04 A.U. agreeing with our measurement of 5·0 A.U. The hydrogen atoms are, of course, fixed to the sides of the chain. In addition to this, one plate taken with a rather high potential on the tube, shows clearly two absorption edges, which, if due to the silver in the film, correspond to spacings of 1·27 A.U. and 0·94 A.U. These correspond fairly well to the planes marked *AB* and *CD* in Fig. 2 below.

The diffuse nature of the reflections given by the 4·2 A.U. and 4·9 A.U. spacings can also be explained from considerations of the smectic state. The planes con-

* G. Friedel, Annales de Physique, Nov. (1922).
† M. de Broglie, *loc. cit.*
‡ Wells, Annales de Physique, Sept., p. 98 (1921).

taining the lengths of the molecules are not crystalline planes, in fact the whole structure is looser than that of a crystal. One can therefore reasonably expect a much greater tolerance in the width of the spacing of these planes than in those of a true crystal.

Microscopic examination of the curds of sodium myristate shows the conical structure described by Friedel for the smectic body. The other substances have

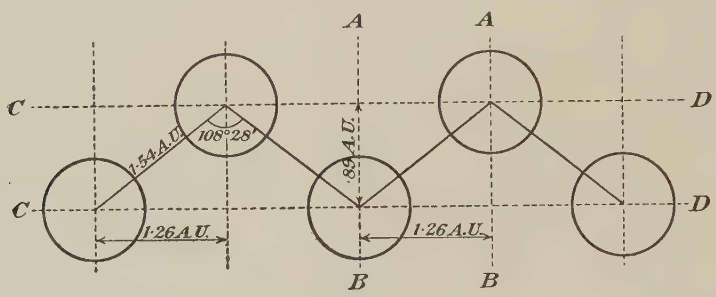


Fig. 2.

not yet been obtained in a state that permits of a satisfactory examination with the microscope.

We are indebted to Professor Tyndall and Professor McBain for suggestions, and to the latter for the specimens. The experiments were carried out in the Physics Department of the University of Bristol with the aid of a grant from the Colston Research Society.

DEMONSTRATION on Intermittent Pressure with Boiling Water. By CHAS. R. DARLING, *F.Inst.P., F.I.C.*

IF a glass tube, open at both ends, and of about 5 mm. bore, be placed in a beaker of briskly boiling water so as to rest on the bottom of the beaker, steam bubbles will be observed to form at the point of contact, causing the water to rise to a definite height in the tube. The column of water thus raised sinks after a time, and then rises again, the rising and falling occurring at irregular intervals. If, however, the tube be narrowed to a bore of about 1 mm. near the top of the water, and widened out considerably just above the water surface, it will then be seen that the phenomenon becomes regular in action. After rising to the height of about 3 cm. in the widened part, the water discharges back again into the beaker, and after a short interval again rises and is discharged, the cycle being repeated indefinitely.

The explanation appears to be that the water is superheated at the points of contact of the tube and beaker, so that the steam produced can sustain a higher pressure of water. When the water reaches the widened part, however, it is cooled and increases in density until the extra steam pressure at the bottom of the tube is overcome, when it discharges completely. The capillary bore slows down the rate of flow in both directions, and so causes the movements to be steady. A separating funnel with open tap and short stem is well suited to the experiment.

It will be observed that the arrangement constitutes a simple heat engine, with source and sink, automatically passing through a regular cycle of operations, and forms a useful lecture demonstration of the conversion of heat into work.

DEMONSTRATION of A Novel Instrument for Recording Wireless Signals. By N. W. McLachlan, *D.Sc., M.I.E.E.*

The device consists essentially of a drum of Swedish iron with an annular recess in which are situated coils of fine wire, the ends of the coils being connected to corresponding slip rings. The periphery of the drum is faced with cast iron rings which are machined to run true to 0·0001 in. A small steel shoe rides on the rings, and side play is prevented by a brass guide-piece with a projection which fits into the annular recess. At each end of the guide-piece a hook is formed, and one of the hooks is connected by a light rod to a duralumin lever pivoted to turn in an horizontal plane. A silver syphon passes through the lever from an ink well overhead and rests lightly on a moving paper tape. To the other side of the lever is attached a strong spring whose tension can be varied, while the remaining hook is attached to a light spring which prevents the shoe from rocking on the drum.

The drum is revolved by a small electric motor, and when a current flows in one of the coils the shoe is attracted to the drum and a large pull is required to prevent relative motion of the two. This pull is used to actuate the syphon lever mechanism so that the transverse movement over the paper tape inscribes the incoming message as a series of rectangles, the tops of which represent the dots and dashes of the Morse code.

The tangential pull on the shoe is many times that calculated from the formula $\mu B^2 A/8\pi$, where μ=coefficient of friction, B=flux density at shoe contact, and A=area of contact. The ratio of the actual to the calculated pull depends on the flux density, and has a maximum value of about 80 for steel or cast iron.

Owing to this phenomenon, which gives amplification, and to the fact that there is no air-gap in the magnetic circuit through the shoe, the instrument is extremely sensitive, and will work at a speed of 150 words a minute with a current of 25 micro-amperes. It will work in a thermionic valve circuit, and for wireless it is fitted with relay contacts, so that incoming messages in Morse may be relayed to a printing or other machine direct.

THE SEVENTH GUTHRIE LECTURE,

Entitled "The Effect of Electric and Magnetic Fields on Spectral Lines." *
Delivered by NIELS BOHR, *University of Copenhagen,*

MARCH 24, 1922.

INTRODUCTION.

IN the characteristic effects on spectral lines observed when radiating substances are exposed to magnetic or electric fields we possess a valuable source of information in regard to the problem of atomic constitution. We have, indeed, in these effects a means of examining in detail the influence of controllable agencies upon intra-atomic processes. This fact has been generally realised by physicists ever since the fundamental discovery of Zeeman† 25 years ago of the characteristic effect of magnetic fields on spectral lines ; and the problem was brought still more within the sphere of interest by the discovery of Stark,‡ about 10 years ago, of the analogous effect of electric fields. Owing to the development of our ideas of electro-magnetic radiation in the interval, however, the way of approach along which explanations of these effects have been sought in connection with theories of atomic constitution has undergone fundamental change. It is the main object of this lecture to expose as clearly as possible the principal features of this development.

I. THE ZEEMAN AND STARK EFFECTS AND THE CLASSICAL THEORY OF RADIATION.

According to the classical theory of electrodynamics, the constitution of the radiation emitted by a system of electrified particles should depend directly on the motion of the particles. In fact, it follows from this theory that every constituent harmonic oscillation in the electric moment of the system must give rise to the emission of a train of waves of frequency coinciding with the frequency of oscillation, and of an intensity depending on the amplitude. Notwithstanding the difficulties of accounting in a simple way for the remarkable empirical laws which govern the frequencies of the spectra of the elements, the characteristic features of Zeeman's discovery obtained, as was shown by Lorentz, an immediate interpretation on this basis. Lorentz§ assumed that each line in the spectrum of the undisturbed atom originates from the motion of an electrified particle performing a harmonic oscillation around a position of stable equilibrium within the atom, under the influence of an attraction directed towards this position and proportional to the first power of the displacement. The component of this displacement in a given direction in space may be expressed by the formula

$$\xi = C \cos 2\pi(\omega_0 t + \gamma) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where the frequency ω_0 is independent of the amplitude of oscillation C.

* In substance this report represents the contents of the Seventh Guthrie Lecture delivered before the Physical Society, March 24, 1922. Due to unavoidable circumstances the publication of this report has unfortunately been delayed until now. N.B. July, 1923.

† P. Zeeman, Phil. Mag., 5, Vol. **43**, p. 226 (1897). *See* also Zeeman's collected Papers on magneto-optical phenomena, Leiden (1921).

‡ J. Stark, Berliner Sitzungsber, Nov. (1913). *See* also Elektrische Spektral-analyse, Leipzig (1914).

§ *See* H. A. Lorentz, The Theory of Electrons, Ch. 3, Leipzig (1909).

Analysing the change in the motion of the particle caused by the presence of an external magnetic field, Lorentz showed that, if the problem is treated on the basis of the ordinary theory of electrodynamics, the original purely harmonic motion is changed in such a way that it can be considered as composed of three constituent harmonic components. One of these is a linear harmonic oscillation parallel to the magnetic field, with a frequency coinciding with that of the undisturbed motion. The two others are circular harmonic rotations in opposite directions in a plane perpendicular to the field, possessing frequencies given by

$$\omega=\omega_0\pm\omega_{\mathrm{H}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where the double sign refers to the two opposite rotations. The expression for ω_{H} is

$$\omega_{\mathrm{H}}=\frac{eH}{4\pi cm}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where H is the intensity of the magnetic field, e and m respectively the charge and mass of the oscillating particle, and c the velocity of light.

This result proved to be in most suggestive agreement with Zeeman's measurements. In fact, in a number of cases spectral lines were observed to be split up into three components, one of which was linearly polarised and appeared in the position of the original line, while the two others were placed symmetrically with regard to the original line and showed circular polarisation in opposite directions. Further, the ratio between charge and mass of the oscillating particles, calculated by means of the Lorentz expression from the displacements of these components, was found to be in close agreement with the value obtained from the deflection of cathode rays in electric and magnetic fields ; and the sense of the polarisation observed for the outer components showed that the charge of these particles, like that of the cathode ray particles, possessed negative sign. This result was generally accepted as a most convincing proof of the electronic theory of matter, and it may certainly be said to have established beyond doubt the conclusion that the origin of these spectra is to be found in the motion of electrons within the atom.

For the general discussion of the Zeeman effect an electrodynamic theorem, first established by Larmor,* is of great importance. According to this theorem the motion of a system of electrons moving in a central field of force will, in the presence of a uniform magnetic field, undergo a change such that the motion of the system, to a first approximation, may be described as a possible motion of the electrons without field, on which is superposed a uniform rotation of the whole system around an axis parallel to the direction of the field, with frequency equal to ω_{H}, as given by formula (3).

This theorem will be seen to include Lorentz's results, since the effect of a superposed uniform rotation on a simple harmonic oscillation is to give rise to a motion of just the type described above. In fact, any elliptic harmonic oscillation may be resolved into a linear vibration in a given direction and an elliptic oscillation in a plane perpendicular to this direction ; and the latter may again be considered as composed of two circular rotations with the same frequency, but opposite directions of revolution. Now a superposed rotation round a given axis will, of

* J. Larmor, Aether and Matter, p. 341, Cambridge (1900).

course, not influence a linear vibration in the direction of the axis ; it will, as regards a circular rotation in a perpendicular plane, simply augment or diminish the frequency by an amount equal to the frequency of the superposed rotation, according as the direction of this rotation is the same as or opposite to that of the original rotation.

However, although a number of spectral lines show Zeeman effects in conformity with the predictions of the theories of Lorentz and Larmor, the lines of many spectra show the so-called " anomalous " Zeeman effects. In such cases the lines are still resolved into components linearly polarised parallel to and components circularly polarised perpendicular to the field, which are arranged symmetrically with respect to the original line, and whose displacements are, at any rate for small intensities, proportional to the field. The number of components and the magnitude of their displacements may, however, differ considerably from what is the case in the " normal " effect. This variation in the character of the Zeeman effect has been found to be intimately connected with the structure of the spectra and the manner in which the lines of these spectra may be arranged in " series." Indeed, according to the rule announced by Preston,* Zeeman effects of the same type are shown not only by the lines of the same spectral series of one element, but also in respect of lines belonging to corresponding series in spectra of different elements exhibiting an analogous structure. Many efforts have been made to explain the appearance of the anomalous Zeeman effect on the basis of the classical theory of radiation. Amongst others may be mentioned the remarkable work of Voigt,† who succeeded in developing formal interpretations of a number of details of the observed phenomena. Nevertheless, great difficulties remained in the attempt to reconcile the anomalous effect with that theory; but it is hardly necessary to enter more closely into this problem here, inasmuch as new difficulties of a fundamental character arise when we try to explain the characteristic effect of electric fields on spectral lines on the basis of the classical theory, even in the case of spectra such as that of hydrogen, where the Zeeman effect is of the normal type.

As is well known, Stark discovered in 1913 that the lines of the hydrogen spectrum showed a resolution into a number of polarised components when radiating hydrogen atoms were exposed to a strong electric field. Regarded from the point of view of the classical theory of the origin of spectra, Stark's results were very surprising. Thus, if an electron performing oscillations round a position of stable equilibrium—as assumed in Lorentz's theory of the Zeeman effect—is exposed to the effect of a uniform electric field, the type of motion should not be changed at all, and the whole effect of the field should consist only in a displacement of the centre of the orbit by an amount proportional to the intensity of the field. Any possible effect on the spectral lines must therefore, on the classical theory, be due to deviations from a central attraction proportional to the displacement exhibited by the forces keeping the electron in its position in the atom. An effect of this kind—which had been considered by Voigt‡ several years before Stark's discovery—should clearly be proportional to the square or higher powers of the intensity of the external fields. In contrast to this expectation an essential feature in Stark's results was that the effect of the electric field on the hydrogen lines was, to a close approximation, directly

* Th. Preston, Nature, 59, p. 224 (1899).

† C . W. Voigt, Magneto-u id Elektrooptik, Leipzig (1908).

‡ W. Voigt, Ann. d. Phys., 43, p. 410 (1991).

proportional to the field. Viewed as a whole the Stark effect, indeed, constitutes a most complex phenomenon which not only differs greatly for different spectra and different series of lines, but which even changes essentially from line to line within one and the same spectral series.

As is well known, the difficulties here alluded to are only a few examples of the breakdown of the ideas of the classical electrodynamics when applied to atomic phenomena. Moreover, as a result of the fundamental researches of Rutherford on the phenomena of radioactivity, we may consider it as proved that the atom consists of a positively charged central nucleus surrounded by a distribution of electrons. On the theory of classical electrodynamics it is clear that such a picture of the atom does not allow of static configurations of stable equilibrium, and that we are forced to assume that the electrons within the atom rotate with high velocities. This gives rise to new difficulties, however, since on the classical theory such motions should be accompanied by a continuous emission of electromagnetic radiation, which would go on until so much energy was radiated that the electrons would fall into the nucleus. It is unnecessary to argue at length the impossibility of explaining the stability of atoms and the emission of spectra consisting of sharp lines on this atomic model, if we confine our considerations to the classical ideas of electrodynamics.

II. Quantum-Theory of Spectra.

A short time before Stark's discovery I ventured to propose a theory of spectra which constitutes a definite break with classical electrodynamics.* This theory is based on considerations of atomic stability and emission of radiation arising from conceptions of the so-called quantum-theory, which was originated by Planck's famous theory of temperature-radiation put forward about 20 years ago, and to the development of which Einstein made fundamental contributions in its earlier stages. The application of the quantum theory to atomic problems rests upon the two following postulates:—

1. Among the conceivably possible states of motion in an atomic system there exist a number of so-called *stationary states* which, in spite of the fact that the motion of the particles in these states obeys the laws of classical mechanics to a considerable extent, possess a peculiar, mechanically inexplicable stability, of such sort that every permanent change in the motion of the system must consist in a complete transition from one stationary state to another.

2. While, in contradistinction to the classical electromagnetic theory, no radiation takes place from the atom in the stationary states themselves, a transition between two stationary states can be accompanied by the emission of electromagnetic radiation, which will have the same properties as that which would be sent out according to the classical theory from an electrified particle executing an harmonic vibration with constant frequency. This frequency ν has, however, no simple relation to the motion of the particles of the atom, but is given by the relation

$$h\nu=E'-E'' \qquad (4)$$

where h is Planck's constant, and E' and E'' are the values of the energy of the atom in the two stationary states which form the initial and final states of the radiation process. Conversely, irradiation of the atom with electromagnetic waves of this

* N. Bohr, Phil. Mag., 26, pp. 1, 476, 857 (1913).

frequency can lead to an absorption process, whereby the atom is transferred back from the latter stationary state to the former.

I shall not enter here on the philosophical problem of the possibility of reaching a satisfactory description of nature by application of such formal postulates, but shall endeavour to show that they allow us to construct a theory which gives a simple and consistent interpretation of spectroscopic phenomena, for the explanation of which the classical ideas of electrodynamics have not proved themselves directly suited.

As the first application we shall consider the so-called "principle of combination of spectral lines," which was brought to light by the researches of Balmer,* Rydberg,† and Ritz‡ on series spectra, and which in recent years has proved to be of general validity in regard to spectra of very different types. According to this principle the frequency of each of the lines of a spectrum may be represented by the formula

$$\nu = T_2 - T_1, \quad \ldots \ldots \ldots \ldots \quad (5)$$

where T_1 and T_2 are two among a manifold of so-called spectral terms. This law, which holds with an accuracy unrivalled in physics, has hitherto resisted any interpretation on the classical ideas, at any rate in a form suitable as a foundation for a detailed discussion of spectroscopic evidence. On our postulates, on the other hand, it is seen that the combination principle can be directly interpreted by identifying the spectral terms with the numerical values of the energy in the possible stationary states, divided by Planck's constant, and by supposing that each spectral line originates from a transition between two of these states.

At first sight this interpretation of the combination principle might be considered as being of a very formal character, since it is not only admittedly at variance with the ideas of classical electrodynamics, but also involves a radical departure from the conceptions on which physical phenomena have hitherto been based. This appears particularly in the assumption that the constitution of the radiation emitted during a process at the outset of which an atom finds itself in a certain stationary state, depends not only on this state but also on that state of the atom which appears as the result of the process. In fact, spectral lines which appear as combinations of various spectral terms with one and the same term are attributed to various possible processes of transition from a certain state of the atom to other stationary states. In the present state of the theory the mode of occurrence of these transitions is considered to be a question of probability, in the sense that an atom in a given stationary state is assumed to possess a certain probability of passing spontaneously in unit time to any of the other stationary states under consideration. This view, which exhibits a marked analogy to the theory of radioactive disintegration, conforms with the assumptions used by Einstein§ in his suggestive deduction of the law of temperature-radiation on the basis of the fundamental postulates stated above.

Notwithstanding the fundamental nature of the departure from the classical electrodynamics which is involved in the quantum theory of spectra, we shall see that it seems possible in a certain sense to regard this theory as a natural generalisa-

* Balmer, Ann. d. Physik, 25, p. 80 (1885).
† J. R. Rydberg, Handl. Akad., Stockholm, 23 (1890).
‡ W. Ritz, Physik. Zeit., 9, p. 521 (1908).
§ A. Einstein, Phys. Zeit., 18, p. 121 (1917).

tion of our ordinary ideas of radiation. Thus it is possible to correlate each of the various processes of transition giving rise to the emission of harmonic trains of waves with one of the various harmonic oscillations which occur in the electric moment of the atom, in such a way that the probability of the occurrence of a transition of a given type is to be ascribed to the presence of a corresponding harmonic oscillation in this moment. This feature of the quantum-theory of spectra, which is termed the "correspondence principle," plays an essential part in the interpretation of the spectral evidence. It may especially be emphasised that it has been possible, by means of this principle, to remove the mystery which has hitherto hung over the application of the combination principle owing to the apparent capriciousness which attends the appearance of the predicted spectral lines.

We shall also see how the correspondence principle has been of great use in developing explanations of the effects of magnetic and electric fields on spectral lines. Before entering into the detailed discussion of these problems, it is necessary to consider briefly the application of the postulates to the simple case of the interpretation of the hydrogen spectrum—which was the starting point of the theory—as well as the main features of the subsequent development of the general principles of the quantum-theory.

III. Theory of the Hydrogen Spectrum.

As is well known, the frequencies of the lines of the hydrogen spectrum may be represented to a high degree of approximation by the simple formula

$$\nu=K\left(\frac{1}{(n'')^2}-\frac{1}{(n')^2}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

For $n''=1$ and $n'=2, 3, 4$ we get a series of lines in the extreme ultra-violet, first observed a few years ago by Lyman* ; for $n''=2$ and $n'=3, 4, 5$ . . . the formula represents the Balmer series in the visible region ; $n''=3$ and $n'=4, 5, 6$ . . . . gives the infra-red series, of which the first two members were observed by Paschen† several years ago ; finally, $n''=4$ and $n'=5, 6$ . . . . correspond with a series quite recently observed in the far infra-red by Brackett.‡

Comparing (6) with formulæ (4) and (5), we may conclude, according to the general considerations of the former section, that the hydrogen spectrum is emitted by an atom which possesses a sequence of stationary states, such that the numerical value of the energy in the nth state is given by $\frac{Kh}{n^2}$. Now, according to Rutherford's theory, the hydrogen atom consists of one electron rotating round a positive nucleus of unit charge. Apart from the small effect due to the change of mass of the electron with velocity, the motion of the atom will be a very simple one, the particles describing simple periodic, elliptic orbits with the centre of gravity at the common focus. For such a motion the frequency of revolution of the particles and the dimensions

* T. Lyman, Phys. Rev., 3, p. 504 (1914).
† F. Paschen, Ann. d. Phys., 27, p. 565 (1908).
‡ F. Brackett, Nature, 109, p. 209 (1922).

of the orbits will, according to the well-known Keplerian laws, be related to the values of the total energy of the system by the simple formulæ

$$\omega=\sqrt{\frac{2W^3}{\pi^2e^4m}}\,,\quad 2a=\frac{e^2}{W}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (7)$$

where ω is the frequency of revolution and $2a$ the major axis of the orbit of the electron, while W is the work necessary to remove the particles to an infinite distance from each other. As before, e and m denote the charge and mass of the electron, while for brevity we have considered the mass of the nucleus as infinitely large. Putting now for the stationary states

$$W=\frac{Kh}{n^2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (8)$$

we obtain for the frequency of revolution and the major axis in these states

$$\omega=\frac{1}{n^3}\sqrt{\frac{2Kh^3}{\pi^2e^4m}}\,,\quad 2a=n^2\frac{e^2}{Kh}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (9)$$

These formulæ give a picture of the formation of the atom by means of a step-wise process in which an electron is bound with emission of radiation in a sequence of stationary orbits of rapidly increasing frequencies and decreasing dimensions, until a state is reached in which the energy of the atom is a minimum and the process of binding is brought to a final stop; this state corresponds to $n=1$ in formula (9). Introducing the empirical values for K as well as e, m and h in (9) we get values for the frequency of revolution and the major axis of the orbit in this "normal" state of the atom, which are of the same order as the values for atomic frequencies and dimensions derived from consideration of the optical and mechanical properties of gases.

On our fundamental postulates there is no question, however, of a closer direct comparison between such formulæ as (9) and formulæ derived on the classical theory of electrodynamics. In particular, we can have no direct comparison between frequencies in the stationary states and frequencies of the spectral lines, as we have assumed that each of these lines corresponds to the radiation emitted during a transition between two states, for which the frequencies of revolution may in general have quite different values. An opportunity for tracing a connection between the spectrum and the motion is offered, however, by the circumstance that the ratio between the frequencies of revolution in two successive stationary states approaches unity when the values of n gradually increase. From (6) we obtain (to a first approximation) for the frequency of the radiation emitted by a transition between two successive states corresponding to large values of n

$$\nu \doteq \frac{2K}{n^3}.$$

Comparing with (9), we find that this expression will coincide asymptotically with the frequency of revolution in the two states if

$$K=\frac{2\pi^2e^4m}{h^3}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (10)$$

As I have shown in the Paper referred to, this condition is actually fulfilled within

the limits of experimental error, if we introduce the empirical values for K as well as h, e and m.*

The connection thus established between the hydrogen spectrum and the quantities describing the model of the hydrogen atom is evidently as close as could be expected, considering the fundamental departure from classical electrodynamics involved in our interpretation of the spectrum. This divergence becomes clear, as soon as we seek an explanation of the effects of magnetic and electric fields on the hydrogen lines; here we meet a problem quite different from that with which we are presented when regarding the matter from the point of view of the classical theory of radiation. A detailed explanation of the Stark effect could not be attained by means of such simple considerations as sufficed for the interpretation of the main features of the hydrogen spectrum; it was found possible nevertheless not only to explain the direct proportionality of the displacement of the components to the intensity of the electric force, but also to account for the absolute magnitude of the effect and the characteristic way in which it varies from line to line in the spectrum.† A closer explanation of the details of the Stark and Zeeman effects, however, demanded the development of methods for the fixation of the stationary states of the atom under the influence of external fields, as well as the formulation of further rules governing the transitions between stationary states and the polarisation of the ensuing radiation. It is of interest to point out that in the case of the Zeeman effect, for the main features of which the classical theory had offered so simple an interpretation, doubt prevailed for a time as to whether this effect could be accounted for at all on the basis of the postulates in the form stated above. In fact it is easily seen that the frequencies of the components into which the lines are split up in the field cannot be represented by a complete combination-diagram of spectral terms. As we shall presently see, a detailed theory of the Stark effect as well as of the Zeeman effect for the hydrogen lines has been established in course of the development of the quantum-theory in recent years.

IV. State Relations for Periodic and Multiple-periodic Motions.

The formal basis of the application of the quantum-theory to atomic problems consists of a number of formulæ which, together with the formula (4), which has often been termed the "frequency relation," allow us to select the stationary states from among the mechanically possible motions of the particles of the atom. These latter formulæ, the so-called "state-relations," may be considered as rational generalisations of the assumption originally used by Planck regarding the possible values of the energy of a system consisting of a particle executing simple harmonic oscillations. Although it is naturally an essential object of the theory to determine the energy of the stationary states, the energy function itself is not, for more complicated systems, well suited to a general formulation of state-relations. A suitable basis for this formulation is found, however, in the so-called integral of action, which plays such an important part in analytical dynamics.

Let us first consider the simple case when the motion of the particles of the atom is simply periodic, independently of the initial conditions. In this case the

* *Cf.* also R. A. Millikan, Phil. Mag., 34, p. 1 (1917).

† N. Bohr, Phil. Mag., 27, p. 506 (1914); 30, p. 394 (1915). *Cf.* also E. Warburg, Verh. Deut. Phys. Ges., 15, p. 1259 (1913).

displacement ξ of each particle in a given direction may be expressed as a function of the time, in the well-known way, as the superposition of a number of harmonic oscillations

$$\xi = \Sigma C_\tau \cos 2\pi(\tau\omega t + \gamma_\tau) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

where ω is the frequency of the periodic motion, and the summation is over all positive integral values of τ. A similar expression holds also, of course, for the component in a given direction of the electric moment of the atom, whose variation with time determines on the classical theory the constitution of the emitted radiation. For such simply periodic systems the stationary states are fixed by a single condition which may be written

$$I = nh \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

where h is Planck's constant, and n a positive integer, the so-called *quantum-number*. The quantity I is defined by

$$I = \int_0 A dt \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

where the integral is the so-called integral of action taken over a complete period of the motion. If the motion is assumed to be governed by the laws of Newtonian mechanics, the integrand A is equal to twice the kinetic energy of the moving particles ($A = \Sigma mv^2$); while, if the modifications demanded by the theory of relativity are taken into account, A is given by the expression

$$A = \Sigma mv^2 \left(1 - \frac{v^2}{C^2}\right)^{-\frac{1}{2}}$$

For the sake of the later discussion it may be noted that this definition of I is identical with the condition

$$I\omega = \bar{A} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

where $\bar{A}$ denotes the mean value of the function A during the motion.

While the relation between I and the total energy ε for different systems may take very different forms, this relation will always obey the simple differential equation

$$\delta E = \omega \delta I \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

where δE and δI denote the difference in E and I for two mechanically possible motions of the system which differ but little from each other.

From (4) and (15) it is seen at once that, in the case of a simple harmonic with oscillator constant frequency ω_0 the motion of which is represented by (1), the equation (12) is equivalent to Planck's well-known relation

$$E = nh\omega_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

We can also easily show that in the case of the hydrogen atom (12) is equivalent to the formula (8), if in this formula the value of K given by (10) is introduced. In fact, noticing that $W = \infty$ for $I = 0$, we get from (7) and (15) by simple integration

$$W = \frac{2\pi^2 e^4 m}{I^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

The general relations (12) and (15) allow us, further, to trace on a broader basis the formal asymptotic connection between the motion of an atomic system and the

spectrum deduced on the quantum-theory, established in the former section in the special case of the hydrogen atom.

Let us consider a transition between two states for which the values of n in formula (12) are equal to n' and n'' respectively. From the frequency relation (4) we get by means of (15)

$$\nu=\frac{1}{h}(E'-E'')=\frac{1}{h}\int_{n=n''}^{n=n'}\omega\delta I \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

If now the numbers n' and n'' are large compared with their difference and consequently the motions in the two states differ comparatively little from each other in frequency and dimensions, we may in (18) consider ω as approximately constant and thus obtain, using (12) also, the asymptotical relation

$$\nu\sim(n'-n'')\omega \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

In the limit, for large quantum numbers, the frequency of the radiation emitted during a transition will consequently coincide asymptotically with the frequency of one of the harmonic constituents of the radiation which, on the classical theory, would be emitted from the atom owing to variation of the electric moment with time; namely, with the frequency of the trains of waves which would result from the harmonic component in the expression (11) for which $\tau=n'-n''$. Now of course there is no question of a gradual approximation of the quantum-theory in the limit of large quantum-numbers to the classical ideas of the origin of radiation. Indeed, just as in the case where these numbers are not large compared with their difference, we assume in this limit that the various harmonic components of the radiation, which would be emitted simultaneously on the classical theory, will originate in quite different processes of transition between different pairs of stationary states. It is exactly this circumstance, however, which leads us to consider the coincidence of frequencies traced in the limit as evidence of a general law which underlies the occurrence of transitions between stationary states.

This law, which has been called the "correspondence principle," states that the occurrence of each transition between two stationary states accompanied by emission of radiation is correlated to one of the constituent harmonic oscillations into which the electric moment of the atom considered as a function of the time can be resolved, to the extent that the occurrence of the transition is conditioned by the presence of the "corresponding" harmonic oscillation. This correlation demands that the probability of the occurrence of a transition shall depend on the amplitude of the corresponding harmonic oscillation of the atom, in such a way that in the limit when the quantum-number is large, the intensity of the emitted radiation in unit time in the mean shall be the same as that which would follow from the classical laws of electrodynamics. A similar connection with the classical theory will be exhibited by the polarisation of the emitted radiation. If, for instance, the corresponding harmonic oscillation in all states of the atom is a linear vibration or a circular rotation, the radiation will have the same constitution as that which on the classical theory would be emitted by an electron executing a harmonic motion of that type.

In the case above considered of a periodic system, the correspondence principle states that the appearance of a transition between two stationary states, in which the quantum-number changes from n' to n'', is conditioned by the presence in the

electric moment of the atom of an $(n'-n'')$th harmonic. This result allows us to throw light on a marked difference between the rules governing the occurrence of transitions between stationary states in the case of a Planck oscillator, on the one hand, and of the hydrogen atom on the other. In Planck's theory of temperature-radiation it is an essential assumption that, as in the classical theory, the frequency of the radiation emitted or absorbed by an oscillator shall be always equal to the characteristic frequency of oscillation. In terms of our theory of spectra this means, as seen from the comparison of (4) and (16), that the oscillator can, in a single step, only pass between two stationary states for which the quantum-number n differs by one unit. On the other hand the interpretation of the hydrogen spectrum necessarily requires that transitions between any pairs of the stationary states of the hydrogen atom shall be possible. On the correspondence principle this remarkable difference is directly accounted for by the fact that the Keplerian elliptic motion of the electron in the hydrogen atom, in contrast to the purely harmonic motion of the Planck oscillator, possesses a complete sequence of upper harmonics.

Owing to the recent development of the quantum-theory it has been possible to establish state-relations not only for simple-periodic systems, but also for systems in which the motion is of the so-called multiple-periodic type. For such systems the displacement of each particle, as well as the variation of the electric moment, may still be represented as a superposition of harmonic oscillations. In contrast to a simple-periodic system the frequencies of these oscillations are not multiples of a single fundamental frequency, but are for a multiple-periodic system of a "degree of periodicity" equal to s, built up as linear expressions of s independent fundamental frequencies in the way shown by the following formula—

$$\xi=\Sigma C_{\tau_1,\ldots,\tau_s}\cos 2\pi\left[(\tau_1\omega_1+\ldots+\tau_s\omega_s)t+\gamma_{\tau_1,\ldots,\tau_s}\right] \quad . \quad . \quad (20)$$

where ω_1 to ω_s are the fundamental frequencies, and the summation is to be extended over all negative and positive values of the integers τ_1 to τ_s.

In such a case the stationary states will be fixed by the s state relations :

$$I_1=n_1h, \ \ldots \ I_s=n_sh, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

where n_1 to n_s are positive integers. The I's are a number of quantities expressing certain properties of the motion, which by analogy with the definition of the quantity I for a periodic system are related to the energy and fundamental frequencies of the motions through the differential equation

$$\delta E=\omega_1\delta I_1+\ldots+\omega_s\delta I_s, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

expressing the energy difference for two mechanically possible motions of the system which differ very little from each other. These relations fix the quantities $I_1 \ldots I_s$ save for an arbitrary constant in each, which, however, is fixed by the condition

$$I_1\omega_1+\ldots+I_s\omega_s=\overline{A}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

where $\overline{A}$ as in formula (14) is the mean value of the function A which appears in the integral of action.

From formula (22) we find that, for the radiation emitted during the transition of the system between two states for which the quantum-numbers in the relations (21)

are given by $n'_1 \ldots n'_s$ and $n''_1 \ldots n''_s$ respectively, the frequency, in the limit where these numbers are large compared with their differences, and where the motions in the two stationary states differ comparatively little from each other, will be given by the asymptotical relation

$$\nu \doteq (n'_1 - n''_1)\omega_1 + \ldots + (n'_s - n''_s)\omega_s. \quad . \quad . \quad . \quad . \qquad (24)$$

According to the correspondence principle we shall consequently assume that a transition between two stationary states of a multiple-periodic system will be dependent on the presence in the expression for the electric moment of the system of a constituent harmonic oscillation for which in (20) we have

$$\tau_1 = n'_1 - n''_1, \ldots \tau_s = n'_s - n''_s \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (25)$$

The establishment of the state-relations for periodic and multiple-periodic systems depends upon the work of a great number of physicists, including Planck himself. It may be of interest to note that a general condition equivalent to (12) was used for the first time by Debye,* and that conditions of a similar type to those in (21) were proposed simultaneously by Wilson† and Sommerfeld.‡

Among the contributions to the later development of the theory we may mention the work of Ehrenfest§ on the adiabatically invariant character of the state-relations. He considers the action on the motion in the stationary states which results from a slow and uniform transformation of the field of force in which the particles of the system are moving, and points out that if the stationary states are fixed by conditions of the type (21) and (22), the effect of this transformation can be described by means of the ordinary laws of mechanics. This so-called " adiabatic principle " constitutes a natural generalisation of the application of mechanics to the description of the motion in the stationary states themselves, which is obviously not at variance with the non-mechanical character of the stability of these states. These problems are discussed in detail in a treatise by the lecturer published a few years ago, in which the correspondence principle was also developed.||

In the application of the theory of multiple-periodic systems to spectral problems the first essential progress was made by Sommerfeld in his interpretation of the fine-structure of the hydrogen lines as revealed when these lines are observed by instruments of high dispersive power, and which is due to the fact that the motion of the electron in the hydrogen atom is no longer strictly periodic when the change of mass of the electron with the velocity is taken into consideration. This work was closely followed by the interpretation of the details of the Stark effect of the hydrogen lines carried out simultaneously by Epstein¶ and Schwarzschild,** and

* P. Debye, Wolfskehl Vortrag Göttingen (1913).

† W. Wilson, Phil. Mag., 29, 795 (1915) ; 31, p. 156 (1916).

‡ A. Sommerfeld, Sitz. der Münchener Akod., p. 425 and 459 (1915).

§ P. Ehrenfest, Proc. Acad. Amsterdam, 16, 591 (1914) ; Phil. Mag., 33, 500 (1914) ; *cf.* also J. M. Burgers, Phil. Mag., 33, 514 (1917).

|| N. Bohr, On the Quantum-Theory of Line Spectra. D. Kgl. Danske Videnskabernes Selskabs Skrifter, 8, iv., 1 (1918) (quoted hereafter as Q.L.S.). For a brief survey of the present state of the theory compare also N. Bohr, Ann. d. Phys., 71, p. 277 (1923). An English translation of this Paper will appear shortly in Proc. Camb. Phil. Soc.

¶ P. Epstein, Phys. Zs., 17, p. 148 (1916) ; Ann. d. Phys., 50, p. 489 (1916).

** K. Schwarzschild, Berliner Sitzungsber, April (1916).

by the work of Sommerfeld* and Debye† on the interpretation of the Zeeman effect for the hydrogen lines. The theories of these effects were completed by the application of the correspondence principle, which allows us to account in detail for the limited number of components observed, as well as for their polarisation and intensities.

The method of representation of the state-relations used by these authors was based on the procedure called "separation of variables" in the integral of action. Quite apart from the more limited applicability of this procedure, the method of representing the state-relations followed here, in which the properties of periodicity of the motion are brought into the foreground, gives us in many cases a more direct insight into the physical meaning of the theoretical considerations. In the succeeding discussions of the application of the general theory we shall therefore not follow the historical order of development, but shall treat the problems in the manner which seems best suited to illustrate the main features of the theory.

V. The Effect of Magnetic and Electric Fields on the Hydrogen Lines.

On the basis of the general considerations in the preceding section we shall now consider in detail the effect of a magnetic and of an electric field on the spectral lines of hydrogen. For this purpose we shall for simplicity neglect the fine-structure of these lines; this is legitimate since the influence on the motion of the electron of variation of mass is very small compared with the effect of external magnetic and electric forces of the intensities used in experiments on the Zeeman and Stark effects. This is clearly shown by the fact that the distance between the fine-structure components of the undisturbed hydrogen lines is much smaller than the displacement of the components into which the lines are resolved in these experiments.

As in Section III., we shall therefore assume the orbit of the electron in the undisturbed atom to be a simple-periodic Keplerian ellipse, for which the frequency of revolution and the major axis are related to the energy as given by (7). Introducing the quantity I defined by (13) we get from (17)

$$E=-W=-\frac{2\pi^2e^4m}{I^2},\ \omega=\frac{4\pi^2e^4m}{I^3},\ 2a=\frac{I^2}{2\pi^2e^2m} \quad \ldots\ldots\ (26)$$

For the stationary states we obtain, therefore, introducing $I=nh$, according to the state-relation (12),

$$E_n=-\frac{1}{n^2}\frac{2\pi^2e^4m}{h^2},\ \omega_n=\frac{1}{n^3}\frac{4\pi^2e^4m}{h^3},\ 2a_n=n^2\cdot\frac{h^2}{2\pi^2e^2m} \quad \ldots\ (27)$$

which are, of course, equivalent to the formulæ (8) and (9) when the value for K is as given by (10). Finally, by the relation (4) we obtain for the frequency of the radiation emitted during a transition between two states for which n is equal to n' and n'' respectively,

$$\nu=\frac{2\pi^2e^4m}{h^3}\left(\frac{1}{(n'')^2}-\frac{1}{(n')^2}\right). \quad \ldots\ldots\ldots\ (28)$$

Effect of a Magnetic Field.

In considering the effect of a magnetic field we shall in the first place assume, according to Larmor's theorem, that the motion of the electron in the

* A. Sommerfeld, Phys. Zs., 17, p. 491 (1916).

† P. Debye, Phys. Zs., 17, p. 507 (1916).

hydrogen atom in the presence of the field differs from a possible motion of the atom without field, in having superposed a uniform rotation around an axis through the nucleus and parallel to the field, with a frequency ω_H given by formula (3). As a consequence of this the displacement of the electron in a given direction is no more represented by an expression of the type (11) holding for a purely periodic orbit, but its motion will contain harmonic components of three different types. One type of component will consist of linear vibrations parallel to the field with frequencies $\tau\omega$, where ω is the frequency of revolution in the periodic orbit which the electron describes in a system of reference which partakes of the superposed rotation impressed by the field. The two other types of harmonic components will be circular rotations in a plane perpendicular to the field with frequencies $\tau\omega+\omega_H$ and $\tau\omega-\omega_H$ respectively; the sense of rotation of the former being the same as, and that of the latter opposite to, the sense of the superposed rotation. Denoting the components of electric moment in directions parallel and perpendicular to the field by ξ and η respectively, we have

$$\xi=\Sigma C_\tau \cos 2\pi(\tau\omega t+\gamma_\tau)$$

and

$$\eta=\Sigma C_{\tau,\pm1} \cos 2\pi((\tau\omega\pm\omega_H)t+\gamma_{\tau,\pm1}) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

The motion of the atom in the field is thus a typical double-periodic motion with the fundamental frequencies ω and ω_H. According to the considerations in the former section the stationary states will therefore be subject to two conditions, which may be written

$$I=nh,\ I_H=n_H h. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

Here I is equal to the quantity defined by (13) when applied to the periodic motion of the atom relative to a system of reference partaking of the superposed rotation, while I_H is equal to 2π times the component M of the angular momentum of the electron around the axis of this rotation. In fact, the change in kinetic energy of the electron due to the superposed rotation is easily seen to be equal, as a first approximation, to $2\pi\omega_H M$. Since the magnetic field does not effect the potential energy of the atom, the energy difference between two neighbouring motions will therefore be expressed by the relation

$$\delta\varepsilon=\omega\delta I+2\pi\omega_H\delta M=\omega\delta I+\omega_H\delta I_H \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

which corresponds to the condition (15). At the same time the condition

$$\omega I+\omega_H I_H=\bar{A}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

which corresponds to (14), is seen to be fulfilled by any motion of the atom in the field. For the total energy of the atom as a function of I and I_H we get from (3) and (26)

$$E=-\frac{2\pi^2e^4m}{I^2}+\frac{eH}{4\pi mc}I_H \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

which, on inserting (30) gives for the energy in the stationary states

$$E=-\frac{2\pi^2e^4m}{h^2}\frac{1}{n^2}+\frac{heH}{4\pi mc}n_H \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (34)$$

It is of interest to point out that the dynamical property of the stationary states expressed by the first of the conditions (30) might have been obtained by a

direct application of Ehrenfest's adiabatic principle. In fact, as shown by Langevin* in his work on atomic magnetism, a slow and uniform establishment of an external magnetic field will, on account of the induced electric forces, affect the motion of a system of electrons revolving in a central field in such a way that the motion at any moment will differ from the original motion only by a superposed rotation possessing the Larmor frequency. On the other hand, it lies in the nature of the problem that the appearance of the second of the quantum conditions (30) can in no way be accounted for by considerations depending on ordinary mechanical and electrodynamical ideas. Indeed the appearance of this condition may be considered as a consequence of the fact that the presence of the external field impresses a new fundamental frequency on the motion of the atom, and thereby calls into play the unknown quantum mechanism governing the stability of the stationary states; the effect being that the energy differences between the various possible states corresponding to the same stationary state of the undisturbed atom will exhibit a relation to the new frequency of the same kind as that between energy and frequency in the stationary states of simple-periodic systems.† In the special case under consideration the additional periodic motion impressed on the atom by the field is of a simple-harmonic character; and it is of interest to note that the second term on the right side of equation (34), which is equal to $n_H \omega_H h$, is quite analogous to Planck's original formula (16) for the possible value of the energy of a harmonic oscillator, with the sole difference that, in accordance with the nature of the problem, n_H can take negative as well as positive values.

Since the maximum value M_0 which the angular momentum P of the electron round the nucleus can take is obviously equal to $I/2\pi$, we see that the second of the conditions (30) is equivalent to the condition

$$M = \frac{n_H}{n} M_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (35)$$

from which it follows at once that the numerical value of n_H can never be larger than n. We must assume that n_H can take any of the values $\pm 1, \pm 2, \ldots \pm n$, while a consideration, which would take us too long to develop here, leads us to the conclusion that no stationary state corresponds to the value $n_H = 0$. For the frequency of radiation emitted by a transition from a state for which $n = n'$ and $n_H = n'_H$ to one for which $n = n''$ and $n_H = n''_H$, we find by (4)

$$\nu = \frac{2\pi e^4 m}{h^2}\left(\frac{1}{(n'')^2} - \frac{1}{(n')^2}\right) + \frac{eH}{4\pi mc}(n'_H - n''_H) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (36)$$

According to the correspondence principle the possibility of such a transition is dependent on the presence in the electric moment of the atom of a harmonic component of frequency $(n' - n'')\omega + (n'_H - n''_H)\omega_H$. Recalling the above analysis of the motion of the electron represented by (29), we see in the first place that, as in the case of the undisturbed atom, there exist possibilities of transitions in which n changes by any number of units. Such transitions, however, will no longer give rise to the usual hydrogen lines, the frequencies of which are given by (28). Instead, as seen from (36), we obtain for each of these lines a number of components corre-

* P. Langevin, Annales de Physique et de Chimie, 5, p. 70 (1905).

† *Cf.* Q. L. S., p. 11.

sponding to the possible simultaneous changes of the quantum-number n_H. These components are of three types. In the first type, which is dependent on the linear harmonic oscillation parallel to the field, n_H remains unchanged, and the components take the same positions in the spectrum as the original lines. On the correspondence principle the radiation corresponding to these components will have the same constitution as the radiation emitted according to classical electrodynamics by an electron performing linear oscillations parallel to the field. In the two other types, which are dependent on the circular harmonic rotations perpendicular to the field, n_H decreases or increases by one unit respectively, and we get for each hydrogen line two components, which are placed symmetrically with respect to the original position of the line, and which, if observed in a direction parallel to the field, will show circular polarisation in opposite directions.

It will be seen that this interpretation of the Zeeman effect for the hydrogen lines exhibits a formal analogy with the original theory of Lorentz, discussed in Section I.; which is really remarkable, when we think of the great divergence between the ideas of classical dynamics and the postulates of the quantum-theory. In the matter, however, of the relative intensities of the components of the Zeeman effect the fundamental departure of the quantum-theory from classical electrodynamics comes to light in an interesting way. According to the classical theory the intensities of these components are determined by the condition that the total radiation of each triplet of components shall not exhibit any sensible resultant polarisation, since the orientation of the atom in the field is not subject to any limitations. On the quantum-theory, on the other hand, where the existence of such limitations is an absolutely essential feature, we should be prepared to find a resultant polarisation of the total light of each triplet, even in weak magnetic fields. Such a polarisation has actually been recorded by various investigators of the Zeeman effect, and it is especially interesting to note that Traubenberg* has, in recent experiments on the hydrogen spectrum emitted by positive rays in a magnetic field, succeeded in observing a resultant polarisation of the kind discussed.

Effect of an Electric Field.

In dealing with the influence of a uniform electric field on the hydrogen spectrum, our first problem will be to examine the effect of the field on the motion of the atom. As in the case of a magnetic field, we meet here with the question of small perturbations in a periodic orbit. In the former case Larmor's theorem allowed us at once to perceive the character of the perturbations; the problem is, however, more complicated in the case of an electric field, which not only produces alterations in the orientation of the orbit in space, but also a continuous change in the shape of the orbit. Nevertheless the problem admits of a simple solution by making use of a general theorem in the theory of perturbations.

Consider a system in which every motion is periodic, and let us imagine that the system is subject to a small external field of force. In this case the motion may be described as a periodic motion, which at any moment differs from a possible motion of the undisturbed system by small quantities proportional to the intensity of the external forces, and which at the same time undergoes slow alterations as regards shape and position of orbit at a rate which is also proportional to these forces.

* R. v. Traubenberg, Naturwissenschaften, 10, p. 791 (1922).

A study of these alterations of motion over long time intervals, which in celestial mechanics are known as "secular perturbations," allows a direct insight into the effect of the external field on the periodic properties of the motion. A fundamental law governing the course of the secular perturbation produced by a fixed field of force is now available in the general theorem referred to above, which states, that, neglecting small quantities proportional to the square of the perturbing forces, the mean value of the potential energy of the system relative to the external field, taken over an approximate period of the motion, will remain unaltered through time intervals long enough for these forces to produce a finite change in the shape and position of the orbit. If we further imagine the external field to be slowly established at a uniform rate, this mean value will, with the same approximation, represent the difference between the total energy of the perturbed system and the original value of the energy of the system before the establishment of the field.*

Reverting to the case of a hydrogen atom perturbed by a uniform electric field, we find by a simple calculation that the mean value of the potential energy of the atom relative to the field is the same as if the electron were placed on the major axis of the orbit at a point dividing the distance between the nucleus and the other focus in the ratio 3 : 1. This point may be called the "electrical centre" of the orbit, and it is an immediate consequence of the general theorem stated above that during the secular perturbation this centre will to a first approximation move in a plane perpendicular to the direction of the external electric force. Now a closer examination of the secular displacement of the electrical centre of the orbit in its plane, which is easily carried out by the simple consideration of the secular changes of angular momentum of the electron round the nucleus due to the external force, shows that the electrical centre performs a purely harmonic, in general elliptical, oscillation, symmetrically placed with regard to an axis through the nucleus parallel to the external force. Moreover the frequency of this oscillation is independent of the shape and orientation of the electron orbit and dependent only on the quantity I, defined for a periodic orbit by (13), which, neglecting small quantities proportional to the external force, will, of course, remain unaltered during the perturbations. Denoting the frequency by ω_F, we have

$$\omega_F=\frac{3I}{8\pi^2 em}F, \qquad (37)$$

where F is the intensity of the electric field.†

Before proceeding to the fixation of the stationary states, we will examine what bearing these results have on the resolution of the motion of the electron into its harmonic components. For this purpose consider the motion relative to a frame of reference which performs a uniform rotation around the axis of the system in the same sense as the rotation of the electrical centre and with a frequency equal to σ. Owing to the harmonic character of the oscillation of the electrical centre this can obviously be described as motion in a periodic orbit, the shape and position of which will vary with frequency equal to $2\omega_F$. Such a motion will be double-periodic with frequencies ω_1 and ω_2, where ω_2 may be taken equal to $2\omega_F$,

* Q.L.S., p. 46.
† Q.L.S., p. 73.

while ω_1 is equal to the mean frequency of revolution of the electron in its approximately periodic orbit, calculated from perihelion to perihelion; which in the system of reference used is obviously equal to the mean frequency of revolution of the electron round the axis. The motion may therefore be considered as a superposition of elliptical harmonic vibrations of frequencies $\tau_1\omega_1+\tau_2\omega_2$, where τ_1 and τ_2 are integers. Returning now to a fixed frame of reference, the motion may be resolved into a sequence of linear harmonic vibrations parallel to the axis with frequencies $\tau_1\omega_1+\tau_2\omega_2$, and two sequences of circular harmonic rotation around this axis with frequencies $\tau_1\omega_1+\tau_2\omega_2\pm\omega_F$. Let us now introduce as fundamental frequencies for the perturbed motion the quantities ω and ω_F, where ω is the mean frequency of revolution of the electron round the axis, and therefore equal to either $\omega_1+\omega_F$ or $\omega_1-\omega_F$, according to whether the electrical centre rotates in the same direction round the axis as the electron itself or in the opposite direction. Denoting the displacement of the electron parallel to the axis by ξ, we obtain consequently

$$\xi=\Sigma C_{\tau,}\tau_F, \cos 2\pi[t(\tau\omega+\tau_F\omega_F)+C_{\tau,}\tau_F], \quad . \quad . \quad . \quad . \quad . \quad . \quad (38)$$

where $\tau+\tau_F$, being equal to $2(\tau_1+\tau_2)$ or to $2\tau_2$, is always an *even* integer. For a displacement η perpendicular to the axis we find similarly:

$$\eta=\Sigma D_{\tau\omega,\,\tau F}\cos 2\pi[t(\tau\,\omega+\tau_F\omega_F)+d\tau,\tau_F] \quad . \quad . \quad . \quad . \quad . \quad . \quad (39)$$

where $\tau+\tau_F$, being equal to $2(\tau_1+\tau_2)\pm1$ or $2\tau_2\pm1$, is always an *odd* integer.

The stationary states of this double-periodic motion will be fixed by two quantum conditions, which can be written in the form

$$I=nh, \quad I_F=n_Fh, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (40)$$

where the quantities I and I_F are related to the total energy of the system and to the action function through the equations

$$\delta E=\omega\delta I+\omega_F\delta I_F \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (41)$$

and

$$\omega I+\omega_F I_F=\bar{A}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (42)$$

which correspond to the conditions (22) and (23). Consider for the moment the especially simple case where the electron moves in a circular orbit in a plane perpendicular to the axis. For such orbits the dependence of E and A on ω, so far as quantities proportional to the external force are concerned, is obviously the same as that holding for a simple Keplerian motion. It follows therefore from (41) and (42) that I_F vanishes in this case, while I coincides with the quantity defined by (13) for a simple periodic orbit. Since ω_F to a first approximation depends only on I, we deduce from this result the following general expression for the energy of the perturbed atom:

$$E=E_0(I)+\omega_F I_F, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (43)$$

where $E_0(I)$ represents the energy of a simple Keplerian orbit expressed as a function of I. From (26) and (37) we find, therefore,

$$E=-\frac{2\pi^2e^4m}{I^2}+\frac{3II_F}{8\pi^2em}F. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (44)$$

The kinematical significance of the second of the quantum conditions (40) follows simply from the general theorem of the adiabatic invariance of the quantum

conditions for multiple-periodic systems. In fact, from a consideration of a slow establishment of the external field, it follows according to the general theorem of p. 291 that the change of the energy of the atom due to the field is equal to ζeF, where ζ is equal to the distance of the electrical centre from a plane through the nucleus perpendicular to the axis. The maximum value ζ_0, in the limiting case in which the orbit degenerates into a straight line parallel to the direction of the field, is equal to $3a/2$, where $2a$ is the major axis of the Keplerian orbit, the dependence of which on I is expressed by the last of the equations (26). Comparing with the second term in (44), we get therefore the simple relation

$$\zeta=\zeta_0 \cdot \frac{I_F}{I} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (45)$$

This relation imposes an obvious limit to the values which n_F can take in the stationary states corresponding to a given value of n. We conclude that n_F can take any of the values $0, \pm 1, \pm 2, \ldots \pm(n-1)$, while the limiting values $\pm n$ have to be excluded on account of the singularity of the corresponding motion.

In connection with the state-relations (40), it may still be of interest to point out that just as in the case when a magnetic field is applied, the second quantum condition ensures a relation between the additional fundamental frequency ω_F, impressed on the atom by the field, and the possible values of the energy of the atom relative to the field, which is completely analogous to formula (16) for the possible energy values of a simple Planck oscillator. This remark illustrates the physical side of the problem of the influence of the field on the stationary states of the atom. It need hardly be emphasised that neither in considering the effect of the magnetic field nor that of the electric field does it suffice to base the treatment on the application of the adiabatic principle to the problem of the slow establishment of the field; this is seen directly from the fact of the entire freedom of orientation in space of the atom in the absence of the field.

Proceeding now to the consideration of the effect of the electric field on the hydrogen lines, we obtain from (40) and (44) for the energy in the stationary states of the atom

$$E=-\frac{2\pi^2e^4m}{h^2}\frac{1}{n^2}+\frac{3h^2F}{8\pi^2em}n\,n_F. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (46)$$

By means of the general frequency-relation this gives, for the radiation emitted by a transition from a state for which $n=n'$ and $n_F=n'_F$ to a state for which $n=n''$ and $n_F=n''_F$,

$$\nu=\frac{2\pi^2e^4m}{h^3}\left(\frac{1}{(n'')^2}-\frac{1}{(n')^2}\right)+\frac{3hF}{8\pi^2em}(n'n'_F-n''n''_F). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (47)$$

According to the correspondence principle, the occurrence of such a transition is conditioned by the presence in the electric moment of the atom of a harmonic component of frequency $(n'-n'')\omega+(n'_F-n''_F)\omega_F$. Comparing this with our analysis of the motion in the field, we are consequently led to infer that each of the spectral components into which the hydrogen lines are split up will show a characteristic polarisation, such that all components for which $(n'-n'')+(n'_F-n''_F)$ is an even integer will show linear polarisation parallel to the field, while components for which

this expression is an odd integer will exhibit a characteristic polarisation in a direction perpendicular to the field. These results are fully confirmed by Stark's experiments; not only can the positions of the observed components for each hydrogen line be accounted for by formula (47) within the limits of experimental error, but also the polarisation of the components is found to conform to the above rules.* Moreover, by theoretical estimation, based on a calculation of the amplitudes of the corresponding harmonic oscillations of the probabilities of transitions giving rise to the various spectral components, it has even been possible to account in detail for the laws of distribution of intensities of the different components, which show great differences from line to line. The latter problem has been treated by Kramers in a Paper which contains a thorough discussion of the problem of intensities of spectral lines in relation to the correspondence principle.†

In view of these results, we may say that the Stark effect for the hydrogen lines, when properly interpreted, reveals every detail of the action of the electric field on the motion of the hydrogen atom. In contrast to the Zeeman effect, however, the image of the motion of the electron which we observe in the spectrum is so distorted that it would hardly have been possible to recognise it on the basis of our ordinary ideas of the origin of electromagnetic radiation. At the same time the fundamental departure of the quantum-theory from classical electrodynamics comes to light in a most striking way in a feature of the effect recorded by Stark.‡ While under usual conditions the components of each hydrogen line exhibit complete symmetry with respect to the position of the original line, a remarkable asymmetry is observed when the spectrum is excited under such conditions that the atom in the main receives impacts from electrons moving in the same or in the opposite direction to that of the electric force. In fact, in the latter condition the components on the long-wave side of the original line are much more intense, or less intense, respectively, than the components on the short-wave side. On the quantum-theory this observation receives immediate interpretation if we assume that under these conditions the probability of the plane in which the electric centre moves being displaced from the nucleus in the same direction as the motion of the impacting electrons, is markedly larger than the probability of this displacement being in the reverse direction. The effect under consideration has often been considered as affording a serious objection to the quantum-theory explanation of the Stark effect.§ We see, however, that, on the contrary, it must be regarded as a most direct evidence of the complete independence of the processes which give rise to the appearance of the various spectral components; and this, according to our postulates, is just an essential feature of the quantum-theory of spectra.||

Effect of Simultaneous Electric and Magnetic Fields.

The considerations applied above allow of direct application to more complicated problems. One such problem is presented when we investigate the simultaneous effect of uniform electric and magnetic fields on the hydrogen lines.

* Q.L.S., p. 77. *Cf.* also Zs. für Phys., 2, p. 446 (1920).

† H. A. Kramers, Intensities of Spectral Lines, D. Kgl. Danske Vidensk. Selsk. Skrifter, 8, 3, 287 (1919).

‡ J. Stark, Ann. d. Physik, 56, 569 (1918).

§ *See* J. Stark, Jahrbuch d. Ra. u. El., 17, p. 161 (1921).

|| *Cf.* N. Bohr, Phil. Mag., 30, p. 402, (1915); *see* also A. Rubinowiz, Zs. f. Phys., 5, p. 331 (1921).

In the case when the two fields are parallel the perturbations of the original periodic motion will be a simple superposition of the perturbations considered above in the case of separate fields, and the stationary states will obviously be fixed by three conditions :

$$I=nh,\ I_H=n_Hh,\ I_F=n_Fh \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (48)$$

where the second condition, which determines the angular momentum of the electron round the axis of the system, as well as the third condition, which determines the position of the plane perpendicular to this axis in which the electrical centre of the orbit moves, are in every respect completely analogous to the additional quantum conditions in the formulæ (30) and (40) respectively. The energy of the atom in the stationary states will consequently be given by

$$E=-\frac{2\pi^2e^4m}{h^2}\frac{1}{n^2}+\frac{ehH}{4\pi mc}n_H+\frac{3h^2F}{8\pi^2em}n\,n_F \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (49)$$

Moreover, it follows directly from the correspondence principle that the effect of the fields will consist in the resolution of every hydrogen line : partly into one set of components polarized parallel to the fields, and placed at the positions of the parallel components of the Stark effect which would appear in the absence of the magnetic field ; partly into two sets of components exhibiting circular polarization in opposite directions and placed symmetrically with respect to the perpendicular components of the Stark effect, in the same way as the circularly polarized components of the usual Zeeman effect are placed with respect to the original line.* This consequence of the theory seems adequately supported by the experiments.†

In many experiments on the Zeeman effect we are concerned with the disturbing effect of small electric fields which possess a component perpendicular to the direction of the magnetic force. This effect may be discussed by considering the resultant motion as a small perturbation of the motion holding in the presence of the magnetic field alone, and the problem may be treated by a method closely analogous to that applied above to the perturbations of a periodic motion. In the present case it may be shown that the electric forces will, in a first approximation, not give rise to the appearance of new fundamental frequencies in the secular changes of the periodic orbit; nor—so far as quantities proportional to the intensity of the electric field are concerned—will this field have any effect on the energy in the stationary states of the atom. The presence of this field, nevertheless, will give rise to the appearance of new harmonic oscillations with amplitudes proportional to the intensity of the field, and with frequencies equal to the sum or difference of two frequencies appearing in the atom when the magnetic field alone is present. On the correspondence principle this will, in addition to probabilities of transitions responsible for the components of the usual Zeeman effect, give rise to small probabilities of the occurrence of new types of transitions. Besides irregularities in the polarisation of the usual components, the electric field may therefore be expected to cause the appearance of new weak components, at distances from the

* Q.L.S., p. 92.
† Garbasso, Phys. Zs., 15, p. 729 (1914).

original lines twice that of the outer components in the normal effect. Such effects have actually been observed.*

Before concluding our consideration of the effect of external fields on the hydrogen spectrum, it may be of interest to characterise, in a few words, the difference between the treatment given here and the method applied in their original investigations on the Stark and Zeeman effects for the hydrogen lines by the authors mentioned in Section IV. These methods were based mainly on the so-called procedure of separation of variables, which in every case inevitably leads to a number of quantum conditions equal to the number of degrees of freedom of the system. As we have seen, however, a treatment of the effect of magnetic and electric fields on the hydrogen lines, adapted to illustrate the physical side of the problem, can be given by using a smaller number of quantum conditions, equal to the degree of periodicity of the motion. The principal objection to the use of a higher number of quantum conditions is that by their use the inherent stability of the spectral phenomena under consideration does not come to light. In fact, these conditions imply the formal fixation of certain properties of the motion of the atom which, in contrast to those fixed by our treatment, are unstable in the presence of external forces which are yet so small that they cannot essentially influence the spectrum. Apart from this, the method of separation of variables has a more limited applicability. This is seen, for instance, on taking into consideration the influence of variability of mass of the electron, which for simplicity has been neglected in the foregoing analysis. The problem of the fine structure of the hydrogen lines can be treated by separation of variables, and so also can the effect of magnetic fields on this fine structure, as was shown by Sommerfeld.† But this is no longer true when we approach the problem of the effect of electric fields on the fine-structure. In this case the motion is of such a complex nature that no set of generalised co-ordinates can be found which allows of a separation of variables. On the other hand, as shown by Kramers,‡ the problem can be successfully treated by considering the motion as a perturbed periodic motion, and examining the periodic properties of the secular perturbations. In this way it is possible to follow theoretically the details in the transformation of the fine-structure of the hydrogen lines which accompanies a gradual increase in the electric field from very small intensities to intensities of the order of magnitude usually applied in experiments on the Stark effect, in which the effect of variability of mass of the electron plays only a very small part. Experiments allowing of a test of these theoretical predictions would be of great interest.

VI. The Effect of External Fields on the Spectra Emitted by Atoms containing Several Electrons.

Notwithstanding their greater complexity the so-called series spectra of many elements show a marked analogy with the hydrogen spectrum. On the quantum-theory of spectra this is accounted for by the assumption that in the stationary states concerned in the emission of these spectra one of the electrons in the atom

* Q.L.S., p. 98.
† A. Sommerfeld, Phys. Zeitschr., 17, p. 497 (1916).
‡ H. A. Kramers, Zeitschr. f. Phys., 3, p. 199 (1920).

moves, at any rate during the greater part of its path, at a distance from the nucleus large compared with the distances of the other electrons. According to this view the spectral lines are emitted by transition processes in which the motion of this electron alone undergoes essential changes, while the orbits of the other electrons coincide quite closely with their orbits in the normal state of the atom. Just as the hydrogen spectrum may be regarded as evidencing a process by which the neutral atom is formed by the binding of the electron by the nucleus, so a series spectrum of this type for another element may be considered as evidencing the last stage of a process in which the atom is formed by the successive capture and binding of electrons in the field of the nucleus. Time will not allow me to enter here into the details of the results it has been possible to attain regarding the general features of atomic constitution by an elaboration of this point of view.* I shall confine myself to showing how these ideas on the origin of series spectra allow us to account for certain main features of the structure of these spectra as well as of the effect of electric and magnetic fields on their lines.

In so far as it remains at distances from the nucleus large compared with the dimensions of the orbits of the inner electrons the force exerted by the rest of the atom on the outer electron will coincide very nearly with the force due to a nucleus of unit charge. In the case in which the outer electron remains always outside the region in which the inner electrons move, its motion may therefore be considered as a Keplerian motion undergoing slow secular perturbations, and of much the same type as the motion in the hydrogen atom under the influence of an external field. In the case when the outer electron penetrates at intervals during its revolution into the inner region, its motion will be composed of a sequence of outer loops, each coinciding closely with part of a Keplerian ellipse and connected with the next by an inner orbital loop in which the motion may differ essentially from a Keplerian motion.

The inherent stability of atomic structure—brought to light so strikingly by experiments on the impact of atoms and free electrons—suggests in the first place that this penetration into the inner region will involve no interchange of energy between the outer electron and the rest of the atom, in the sense that for one and the same electron orbit the various outer loops will coincide closely with parts of ellipses corresponding to the same value of the energy of the hydrogen atom. Moreover, the general central symmetry of the electronic arrangement in the nuclear atom suggests that consecutive loops will as a first approximation have the same shape and be spaced at equal angles to each other in the orbital plane. This means that the motion may be considered as a plane periodic motion on which is superposed a uniform rotation in its plane, a description which may be assumed to hold in a first approximation for the outer orbit, whether it penetrates into the inner region or not. On this motion may be superposed again a slow precession of the orbital plane around the invariant axis of angular momentum of the atom.

The resolution of a motion of this type into its harmonic components may be simply effected in the following way. In a system of reference partaking of the

* *Cf.* The Theory of Spectra and Atomic Constitution, Camb. Univ. Press (1922), containing three essays of which the first two deal in a general way with the problems considered in the former sections of this lecture, while the third gives a detailed discussion of the theory of the constitution of atoms. *See also* my Paper Linienspectren und Atombau (Ann. d. Physik, 71, p. 229, 1923) which contains a more detailed account of the interpretation of spectra with complete reference to the literature.

rotation of the orbit in its plane and the precession of the orbital plane, the motion of the electron will be composed of a sequence of elliptic harmonic oscillations of frequencies $\tau\omega$, where τ is an integer and ω the frequency of revolution. It is seen that each of these oscillations, as a consequence of the uniform rotation of the orbit in its plane, will be resolved into two circular harmonic rotations in opposite directions, with frequencies $\tau\omega\pm\omega_R$, where ω_R is the frequency of the orbital rotation. As in the consideration of the Zeeman effect, each of these will, on account of the precession of the orbital plane, be resolved into a linear harmonic vibration parallel to the fixed axis with unaltered frequency, and two circular harmonic rotations in opposite directions with frequencies increased or diminished by the frequency of precession. Denoting this frequency by ω_P we find consequently for the displacement of the outer electron in directions parallel and perpendicular to the invariant axis respectively,

$$\xi=\Sigma C_{\tau,\pm1}\cos 2\pi[(\tau\omega\pm\omega_R)t+\gamma_{\tau,\pm1}],\quad \eta=\Sigma D_{\tau,\pm1,\pm1}\cos 2\pi [(\tau\omega\pm\omega_R\pm\omega_P)t+\delta_{\tau,\pm1,\pm1}]. \quad . \quad . \quad . \quad . \quad (50)$$

By analogy with the general theory of the state-relations for periodic and multiple-periodic systems we shall assume that the motion of the outer electron in the stationary states is determined by a set of conditions which may be written

$$I=nh,\ I_R=n_R h,\ I_P=n_P h \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (51)$$

where the quantities I, I_R and I_P are correlated with the fundamental frequencies in the triple-periodic motion of the outer electron by the relation

$$\delta E=\omega\delta I+\omega_R\delta I_R+\omega_P\delta I_P, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (52)$$

which refers to two states of the atom for which the orbits of the inner electrons retain their shape and relative configuration while the shape of the orbit of the outer electron as well as its orientation relative to the inner orbits differ slightly.

Since the perturbations of the outer orbit from a Keplerian ellipse occur almost entirely in the region close to perihelion, where the electron remains only a small fraction of the period required to traverse an orbital loop, this period will to a high degree of approximation be equal to that required for the description of the Keplerian ellipse of which the outer loop forms a part. With this approximation we have therefore

$$\delta E=\omega\delta I_0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (53)$$

where I_0 is the quantity defined by (13) as applied to this ellipse. Comparing (52) with (53) we may therefore write

$$I_0=I+\Phi(I_R, I_P), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (54)$$

where Φ is a function of I_R and I_P subject to the relation

$$\omega\delta\Phi=\omega_R\delta I_R+\omega_P\delta I_P. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (55)$$

from which it follows that the ratios ω_R/ω and ω_P/ω, with the approximation under consideration, are independent of I. For the dependence of the work W required to remove the outer electron from the atom we obtain now in accordance with (26)

$$W = \frac{2\pi^2 e^4 m}{I_0} = \frac{2\pi^2 e^4 m}{(I+\Phi)^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (56)$$

Writing for simplicity $E = -W$ we get with reference to (51) for the energy in the stationary states

$$E = -\frac{2\pi^2 e^4 m}{h^2} \cdot \frac{1}{[n+\alpha(n_R, n_P)]^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (57)$$

where α stands for Φ/h. This formula accounts at once for the series structure of the spectra under consideration. In fact, the empirical expressions for the spectral terms within each series have in a first approximation just the same form as (57), if α is taken as a constant characteristic of each series, and n is given a sequence of consecutive integral values. In other words, each series of spectral terms may be correlated to the stationary states corresponding to a sequence of integral values of the "principal" quantum number n and constant values of the "subordinate" quantum integers n_R and n_P.

So far the considerations have been independent of our special assumptions as regards the deviations of the orbit of the outer electron from a Keplerian orbit and of the dynamical significance of the symbols I_R and I_P. Applying a relation analogous to (32) it is simply shown, however, that with the assumptions used as to the character of the perturbations, $2\pi I_R$ will be the angular momentum of the outer electron around the nucleus, while $2\pi I_P$ will be the resultant angular momentum of the whole atom around the invariant axis. On the basis of this result a detailed classification of the manifold of terms has been obtained correlating each series of "terms" in extended sense with a given value of n_R, while the complex structure of these terms (doublets, triplets, &c.) is accounted for by correlating each term "component" to a given value of n_P. This classification, which is due principally to Sommerfeld, has received most convincing support from the application of the correspondence principle. According to this principle the appearance of a transition from a stationary state characterised by n', n'_R, n'_P to another state characterised by n'', n''_R, n''_P, is conditioned by the presence in the motion of a constituent harmonic vibration of frequency $(n'-n'')\omega+(n'_R-n''_R)\omega_R+(n'_P-n''_P)\omega_P$. In view of the analysis of the motion expressed by (50) we conclude therefore that while for a given transition n can change by an arbitrary number of units, n_R can only increase or decrease by a single unit, and n_P remains unchanged or changes by one unit. The classification of the empirical manifold of spectral terms has indeed been effected in such a way that these theoretical rules of combination are obeyed.

Proceeding now to the effect of electric and magnetic fields on series spectra, we find that the application of the same principles which have guided us in examining these effects in the case of the hydrogen spectrum leads us to a number of theoretical predictions which have been found to be fulfilled to a large extent.

In the case of the electric field we meet at once with a typical difference from the conditions encountered in hydrogen. Owing to the periodic character of the electronic orbit in hydrogen, the external field produces a finite change in the shape and position of the orbit, due to an accumulation of the effects of the secular perturbations. In the motion involved in the series spectra of other elements, on the

other hand, we have in the undisturbed atom to do with an electronic orbit which is continuously undergoing regular changes as regards its position in space, of a type that limits a cumulative effect of the perturbations to time intervals of the same order of magnitude as the periods characteristic of these changes of position. As long as these periods are short compared with the period of the changes which the same field would produce in a purely Keplerian orbit of the same dimensions, the character of the motion will undergo only small periodic changes, and notably there will appear no secular perturbations chacterised by a new frequency proportional to the first power of the external forces. In the case of the series spectra of elements other than hydrogen there will, therefore, be no question of a splitting up of the spectral lines into components with a displacement proportional to the field, at any rate so far as the spectral terms involved differ from the hydrogen terms with the same principal quantum number by an amount which is large compared with the effect of the same field on the hydrogen terms. In this case any resolution or displacement of the spectral lines will be proportional to the second power of the electric forces, and the effect will be the smaller, the more the spectral terms involved deviate from the hydrogen terms, which deviations, according to (55) and (56), indeed give a measure of the frequencies of the changes of position of the orbit in space.

These theoretical expectations are completely confirmed by the experiments of Stark and other investigators, which have shown that an effect of the electric field on the lines, of the same order of magnitude as exists in hydrogen, occurs only for lines for which at least one of the two spectral terms involved coincides closely with the hydrogen term of the same quantum number; while for lines where both terms deviate considerably from hydrogen terms, the effect is very small, if measureable at all.

The problem of the influence of electric fields on the spectral lines can be followed in great detail, both as regards the theoretical predictions as well as their confirmation by experiment. It will, however, carry us too far here to enter more closely into these questions. I shall, however, mention one very important feature brought out by Stark's experiments, namely, the production of new combination lines under the influence of the field. This phenomenon receives an immediate explanation on the theory. In fact, although, as mentioned, the electric field does not change the type of the motion of the electron in a first approximation, there will nevertheless, on account of the perturbations, appear new constituent harmonic oscillations in the motion, with amplitudes proportional to the electric forces, and with frequencies equal to the sums or differences of frequencies belonging to harmonic constituents present in the undisturbed motion. Owing to these new oscillations, which are analogous to the "combination tones" well known in acoustics, the atom will acquire, apart from the transitions giving rise to the usual spectral lines in the presence of the field, possibilities of new transitions giving rise to new spectral lines, the frequencies of which will be equal to the sum or difference of the frequencies of lines which appeared in the undisturbed spectrum.* As far as experimental evidence is available, these expectations are fulfilled both as regards the position of the new lines, and as regards their intensities estimated on the correspondence principle. The observation of such "true" combination lines has been

* *Cf.* Q.L.S., pp. 36 and 108.

generally considered among the strongest supports of the combination principle, although at the same time the apparent capriciousness of their appearance threw a veil of mystery over the application of this principle. To-day it is seen, however, that the quantum-theory has not only afforded a formal interpretation of the combination principle, but that it has also contributed materially to the clearing up of the mystery surrounding its applications.

Considering next the effect of a uniform magnetic field we find that the application of the laws of electrodynamics, together with the correspondence principle, leads to very simple deductions. In fact, quite independently of the character of the motion of the electrons in the absence of the field, we should expect from Larmor's theorem that the effect of the field would consist simply in the superposition of a uniform rotation of the whole atom around an axis parallel to the field. Just as in the case of hydrogen, the superposed rotation will give rise to the appearance of a new quantum condition, to the effect that only those orientations of the atom relative to the field are possible in which the component of the total angular momentum of the atom parallel to the field is equal to an integral multiple of $h/2\pi$. Moreover, on the correspondence principle, the effect of the superposed rotation on each of the harmonic constituents in the motion of the atom, in the absence of the field, would involve the resolution of each line into a normal Lorentz triplet.

These theoretical expectations are, however, as already mentioned at the beginning of this lecture, only partly fulfilled. While all spectra consisting of single lines, indeed, show the normal effect, the series spectra of more complicated types exhibit, as is well known, the so-called anomalous Zeeman effect. On the correspondence principle this may be considered as proving that, in contrast to the laws of classical electrodynamics, the magnetic field will, for spectra of such types, not only affect the motion of the atom as a whole, but will also directly influence the interplay between the various electrons in the atom. This is especially clearly shown by the way in which the anomalous Zeeman effects, as first observed by Paschen and Back, are gradually transformed by increasing field intensity; as well as by the appearance observed by the same authors of new components in the complex structure of the series lines in the presence of the magnetic field.* The latter phenomenon may be considered as the complete analogue of the appearance of new series lines in the presence of external electric fields. At the same time these effects show clearly that the magnetic field does not directly influence those properties of the motion which are fixed by the principal quantum number n as well as by the subordinate quantum-number n_R. This is also satisfactory, since not only the approximately Keplerian character of the orbital loops but also the rotation of these loops in their plane, depends only on the simple assumption that the effects on the motion of the outer electron exerted by the rest of the atom conform approximately to that of a central field of force. On the other hand the properties of the motion fixed by the quantum number n_p involve directly the dynamic character of the configuration of inner electron orbits, and may be considered as representing primarily the finer interplay of the outer electron and the atomic residue. In view of the above considerations the anomalous Zeeman effect suggests that these features of the interplay cannot even in a first approximation be described by the

* F. Paschen und E. Back, Ann. d. Phys., 39, p. 897 (1912), and Physica, 1, p. 261 (1921).

laws of classical electrodynamics. Indeed, only on this view does the breaking down of Larmor's theorem in these cases seem intelligible ; and it is therefore most satisfactory that the other features in regard to the complex structure of series spectra also point definitely to this conclusion.*

A most suggestive clue to the examination of this problem may be considered as afforded by Preston's rule, which was referred to at the beginning of this lecture, as well as by the rules first established by Runge concerning the simple numerical relations between the displacements of the components of the anomalous effects and those of the normal Zeeman effect. In this respect a step of great significance has recently been made by Landé, who has succeeded from the empirical rules in deriving general laws governing the way in which a given spectral term, under the influence of a magnetic field, is split up into a number of term-components, as well as the manner in which these components combine with each other to give rise to the observed resolution of the spectral lines.† It is to be hoped that these beautiful results will help to clear up the still unsolved secrets of electron interplay in the atom. Ingenious and suggestive attempts have already been made in this direction, but a satisfactory solution is hardly yet in sight. As indicated above, such a solution will presumably demand a still greater departure from the classical conceptions, even though it may be expected to conform with the general ideas regarding the stability of atoms and the radiation emitted by them, the illustration of which forms the principal purpose of this lecture.

* *Cf.* N. Bohr, Ann. d. Phys., 71, p. 277 (1923).
† A. Landé, Zeitschr. f. Phys., 5, p. 231 (1921), and 15, p. 189 (1923).

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